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# Numerical study of unsteady, thermally-stratified shear flows in superposed porous and pure-fluid domains

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ABSTRACT

In this paper we report on numerical simulations of temporally evolving, thermally stratified shear flows in superposed porous and pure-fluid domains. In particular, we study two different types of flows, namely, pressure-driven channel flows and unforced shear layers. Our study is based on a recently developed thermo-mechanical model for flows in the domains of interest. This model follows a mixture-theoretic approach, according to which the medium's porosity is introduced as a concentration parameter, thus allowing the derivation of a single set of equations that is valid simultaneously in both domains. For the types of flows examined herein, our simulations predict the formation of spanwise vortical structures (rollers) at the porous–pure fluid interface. These rollers grow in time thereby inducing fluid circulation inside the porous medium. For the case of pressure-driven channel flows, our simulations further predict the development of plumes of hot and cold fluid due to convective instabilities that interact with the rollers. In the case of unforced shear layers, unstable stratification accelerates the growth of the rollers, which soon start to merge, and enhances fluid mixing. By contrast, the effect of stable stratification is exactly the opposite. Herein we discuss in detail the temporal evolution of the predicted flow structures and the interactions between them, as well as the mechanisms that induce thermal non-equilibrium inside the porous medium.



## 1. Introduction

In this paper, we report on numerical simulations of unsteady, thermally-stratified flows in superposed porous and pure-fluid domains. More precisely, herein we study the characteristics and temporal evolution of two types of such flows. The first type consists of pressure-driven (forced) channel flows, whereas the second one consists of unforced and semi-bounded shear layers. The study of these types of flows is motivated by their occurrence in numerous industrial applications, such as heat-transfer equipment, air-filtration systems, electronic cooling, as well as in various environmental phenomena.

As regards forced convection in a channel, due to its technological applicability, it has been studied theoretically by various researchers in the past. For example, the stability characteristics of constant-density flows in channels consisting of porous and pure fluid layers has been studied in [1,2]. Also, the stability of forced convection in such channels and in a variety of settings has been examined in [3–8], whereas the stability of gravity driven flows of this type has been studied in [9,10].

In addition to these theoretical studies, a considerable amount of work has been devoted to the numerical treatment of such flows. For example, numerical solutions of steady-state flows have been presented in [11–16], and others. However, numerical results for unsteady flows in the domains of interest are particularly scarce, a noticeable exception being the simulations reported in [17] of constant-density turbulent flow over a permeable wall. Therein, the authors considered both the discrete and the continuum-medium representation of the wall. In particular, the wall was treated alternatively as an array of fixed cubes and as a continuum porous medium; comparisons between the numerical predictions according to these two different representations have also been provided by the authors. Also, more recently, the authors of [18] presented simulations of turbulent heat transfer at low Prandtl numbers in a channel partially filled by an array of fixed cubes by modelling the fluid temperature as a passively convected scalar.

As mentioned above, in addition to simulations of forced convection in a channel, herein we also report on numerical simulations of unforced, semi-bounded shear layers under thermal stratification. This work is the continuation of an earlier numerical study of ours on isothermal shear layers at a porous medium–pure fluid interface [19]. According to that study, the onset and growth

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of the shear-induced Kelvin–Helmholtz instability at the interface leads to the formation of spanwise vortical structures that are commonly referred to as *rollers*. These rollers are similar to the ones that are formed after the roll-up of a Kelvin–Helmholtz vortex sheet in plane mixing layers; see, for example, the classical experimental study [20], the numerical simulations of plane mixing layers presented in [21,22], and the theoretical analysis on the roll-up of a vortex sheet provided in [23,24]. As in the case of plane mixing layers, the rollers that are developed at the porous medium–pure fluid interface begin to merge soon after their formation. The process of roller formation and pairing causes fluid entrainment from the pure-fluid region to the shear layer and significant fluid recirculation inside the porous layer. Also, according to the numerical predictions in [19], after the completion of the pairings of rollers, the shear layers grow in a self-similar manner. In the present paper we extend this work by studying the effects of both stable and unstable stratification on the evolution and characteristics of shear layers at a porous medium–pure fluid interface.

It is worth mentioning that one of the motivations for the study of such shear layers is the problem of flow over vegetation canopies; such structures are typically represented as a permeable medium with high porosity, see [25,26]. Initially it was proposed that this type of flow resembles a boundary layer and that the canopy essentially plays the role of a rough wall. However, in later studies, it has been argued that flow over vegetation canopies resembles a turbulent mixing layer and not a boundary layer, which implies significant fluid recirculation inside the porous medium; see, for example [25–28], as well as the review article [29]. Indeed, as it will be shown below, our simulations predict the development of a shear layer that grows with time, thus corroborating the mixing-layer analogy, under both stable and unstable thermal stratification.

In our numerical study we employ a thermo-mechanical model for coupled porous media–pure fluid flows recently developed by the authors [30]. This model adopts the so-called one-domain approach, *i.e.*, it consists of a single set of governing equations that is simultaneously valid in both the porous and the pure-fluid domains. Further, it is based on the mixture-theoretic formalism for immiscible mixtures presented in [31,32], according to which the solid matrix of the porous medium and the fluid phase are treated as two interacting thermodynamic continua that are in thermal and mechanical non-equilibrium with each other. Moreover, according to this formalism, constitutive expressions for all dissipative and relaxation phenomena are derived via exploitation of the constraints imposed by the entropy inequality of the two-phase mixture. It is worth mentioning that various other thermodynamically consistent models for porous-media flows have appeared in the literature; the interested reader is referred to [33–38] and references therein.

It should be added that in this paper we are concerned with temporally-evolving, two-dimensional flows in rectangular domains consisting of a porous layer and a superposed pure-fluid layer. To the best of our knowledge, such numerical study has not yet appeared in the literature. This particular set-up is chosen because it is easy to implement and because it can provide important insight and deeper understanding of the underlying physical processes in the flows that occur in the afore-mentioned technological applications and environmental phenomena. To this extent, in the discussion of the numerical results, particular emphasis is placed on: (i) the effects of thermal stratification, (ii) the evolution of the flow structures at the macroscopic interface between the porous and pure-fluid layers, and (iii) the mechanisms that induce thermal non-equilibrium between the solid matrix and the fluid phase inside the porous medium. Finally, it is worth mentioning that even though three-dimensional effects are not taken into account, our simulations can still be used for the analysis of the flows of interest, at least in the early stages of their evolution.

This article is organized as follows. In Section 2 we present the governing set of equations and we briefly describe the algorithm that we employ for its numerical treatment. In Section 3 we explain the parametrization that we have used for all transport coefficients entering the governing equations. Next, in Section 4, we present and analyze results from numerical simulations of isothermal, pressure-driven flow in a channel consisting of a porous layer and a superposed pure-fluid layer. Subsequently, in Section 5 we describe results from numerical simulations of pressure-driven channel flows with the same configuration but with the channel walls being kept at different temperatures (forced convection in a channel). Finally, in Section 6 we present and discuss our simulations of unforced, semi-bounded shear layers under thermal stratification.

## 2. Mathematical formulation and numerical aspects

Let $\Omega \subset \mathbb{R}^2$ be a domain consisting of a porous layer $\Omega_\mathrm{p}$ and a superposed pure-fluid layer $\Omega_\mathrm{f}$. The porosity distribution $\phi(\boldsymbol{x})$ is introduced as a concentration parameter that measures the density of volume occupied by the fluid. As such, $\phi(\boldsymbol{x}) < 1$ for $\boldsymbol{x} \in \Omega_\mathrm{p}$ whereas $\phi(\boldsymbol{x}) = 1$ for $\boldsymbol{x} \in \Omega_\mathrm{f}$. The axiomatic definition of porosity can be found, for example, in [39]. The interface between the porous and pure-fluid layers is denoted by $S_\Omega$. The skeleton (or matrix) of the porous material is assumed to be a rigid solid of zero velocity and constant mass density. Also, the fluid phase is assumed to consist of a single Newtonian and isotropic fluid substance. Finally, the two co-existing phases in the porous domain $\Omega_\mathrm{p}$ are assumed to be, in general, at non-equilibrium with each other.

Following standard notation, let $\rho, \boldsymbol{u} = (u, v), p$, and $T$ denote, respectively, the fluid's density, velocity vector, dynamic pressure, and temperature. These and all other variables and physical parameters appearing herein are understood to have been non-dimensionalized by appropriate reference values. Also, let $p^0$ denote the fluid's thermodynamic pressure. Herein we are concerned with low-Mach number flows and, therefore, $p^0$ is function of time only, *i.e.*, $p^0 = p^0(t)$. Further, we assume an perfect-gas equation of state, so that $p^0 = \rho T$. Finally, let $T_\mathrm{s}$ denote the temperature of the solid matrix.

According to the thermo-mechanical model of [30], the mass, momentum and energy balance laws for the fluid phase, as well as the energy balance law for the solid matrix read, respectively,

$$\phi \frac{\partial \rho}{\partial t} + \nabla \cdot (\phi \rho \boldsymbol{u}) = 0, \tag{1}$$

$$\phi \rho \frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} + \phi \nabla p = \frac{1}{Re} \nabla \cdot (\phi \mu \boldsymbol{V}) - \boldsymbol{\beta} \boldsymbol{u} - Ri\, \phi \rho \hat{\boldsymbol{y}}, \tag{2}$$

$$\phi \rho c_p \frac{\mathrm{d}T}{\mathrm{d}t} - \phi \frac{\gamma - 1}{\gamma} \frac{\partial p^0}{\partial t} = \frac{1}{RePr} \nabla \cdot (\phi k \nabla T) + h(T_\mathrm{s} - T), \tag{3}$$

$$(1 - \phi) \rho_s c_\mathrm{s} \frac{\partial T_\mathrm{s}}{\partial t} = \frac{1}{RePr} \nabla \cdot ((1 - \phi) \boldsymbol{k}_\mathrm{s} \nabla T_\mathrm{s}) - h(T_\mathrm{s} - T). \tag{4}$$

In the above equations, $\boldsymbol{V}$ represents the deviatoric part of the fluid's deformation tensor, i.e. $\boldsymbol{V} = (\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^\mathrm{T}) - \frac{2}{3}(\nabla \cdot \boldsymbol{u})\boldsymbol{I}$, $\boldsymbol{I}$ being the identity matrix. Also, $c_p, \gamma$ and $k$ denote the fluid's specific heat under constant (thermodynamic) pressure, Poisson's ratio and conductivity, respectively. Further, $c_\mathrm{s}$ and $\boldsymbol{k}_\mathrm{s}$ denote the specific heat and conductivity of the solid matrix. Finally, $\boldsymbol{\beta}$ and $h$ represent the interphasial drag and heat transfer parameters, respectively. It should be mentioned that since the solid matrix is in general anisotropic, $\boldsymbol{k}_\mathrm{s}$ and $\boldsymbol{\beta}$ are second-order tensors. We also mention that herein the Richardson number is defined as $Ri \equiv g\, l\, u_\mathrm{ref}^{-2}$, with $g$ being the gravitational acceleration.