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Mass transfer induced slip effect on viscous gas flows above a shrinking/stretching sheet

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ABSTRACT

Viscous gas flows above linearly shrinking and stretching sheets with mass transfer are studied via theoretical analysis. Effect of mass-transfer induced slip at a moving surface on gas flows, which has not been considered in all previous studies, is systematically investigated. Net mass-transfer on a moving surface is demonstrated by a newly developed slip flow model (Wu, 2014) to introduce a gas slip velocity component in addition to the slip velocity component due to velocity shearing considered in the original 1st or 2nd order slip flow models. Our results show that mass suction induces a slip velocity against the sheet motion. The competition between flow driven effects of mass-suction induced slip and sheet motion significantly expands the available solution space, and adds to the solution space one more solution sub-region, which is absent when mass-suction induced slip is not considered. Mass-suction induced slip may even achieve a dominant flow driven role by totally reversing the flow direction of adjacent gases to flow against the sheet motion. Mass-injection induced slip enhances the flow driven effect of the moving sheet.

1. Introduction

Viscous gas flows driven by a shrinking or stretching sheet with mass suction or injection exist in many industrial applications such as plastic sheet extrusion, paper production, fiber spinning, etc., to mention just a few. The related viscous gas flow problems have been extensively studied in the past four decades due to their industrial relevance [1–9]. Miklavcic and Wang [1] were the first to obtain an exact similarity solution for gas flows over a linearly shrinking sheet with mass suction, and their results suggest that dual solutions exist in a certain range of mass suction rate. The shrinking sheet driven flow problem was extended afterwards to include power-law shrinking velocity [2], and MHD flows [3,4], etc. Gas flows over a stretching sheet have been studied for both cases without [5] and with mass transfer [6].

One aspect distinguishing micro/nanoscale gas flows from their macroscale counterparts is that wall-slip effect usually becomes so vital at micro/nanoscales that its negligence may lead to predictions unacceptably deviated from physical reality. The Navier–Stokes equation, together with adequate slip velocity boundary conditions that properly incorporate gas molecule and wall interaction kinetics, has been demonstrated to provide accurate results for micro/nanoscale gas flows [10].

Maxwell's 1st order slip flow model [11] was employed by Wang [7] in the study of stretching sheet problem. Fang et al. [8,9] considered wall-slip effect on shrinking/stretching sheet driven flows by adopting a 2nd order slip flow model developed by Wu [10], and found that the flow solutions highly depend on the 1st and 2nd order slip coefficients.

We very recently derived a slip velocity boundary condition for rarefied gas flows above a moving surface with net mass transfer from kinetic theory [12]. The wall-slip velocity can be obtained by equating the total tangential momentum transfer rate of gas molecules at the wall to the viscous wall shear stress [10–13]. When mass is sucked from or injected into the flow domain through a moving wall, it is usually unavoidable for the gas molecules leaving/entering the gas flow domain to have a non-zero average tangential velocity. Consequently, mass transfer contributes to the total tangential momentum transfer rate of gas molecules at the moving wall, and in turn introduces an additional gas slip velocity component linearly proportional to the mass transfer rate and the average tangential velocity of gas molecules leaving/entering the gas flow domain through the moving wall [12]. The mass transfer induced slip velocity component is very different from the previously studied gas slip velocity component due to velocity shearing [10,11,13], and may have a magnitude comparable to or even larger than the latter.

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Mass-transfer induced slip effect has not been considered in all previous studies of viscous flows over shrinking/stretching sheets [1–9]. In this paper we theoretically study viscous gas flows over both shrinking and stretching sheets with mass transfer by fully considering the effect of aforementioned mass transfer induced slip. Exact similarity solutions are obtained for both shrinking and stretching sheet driven flows. Our results show that mass-transfer induced slip has a non-negligible effect and thus has to be considered in gas flows driven by a moving sheet with mass transfer.

2. Theoretical models

The viscous gas flows under consideration are steady state 2-D flows driven by an underneath sheet moving horizontally at a speed u_w (Fig. 1). Similar gases are sucked from or injected into the gas flow domain through the moving surface at a wall-normal speed v_w. The continuity and Navier–Stokes equations governing the flows are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{3}$$

where u is the velocity in x direction, v is the velocity in y direction, ρ is the density, p is the pressure, and ν is the gas kinematic viscosity, respectively. The corresponding boundary conditions are

$$u(x,0)=u_w+u_{slip}, \tag{4}$$

$$v(x,0)=v_w, \tag{5}$$

$$u(x,+\infty)=0, \tag{6}$$

where $u_w=\pm ax$ is the shrinking (negative sign) or stretching (positive sign) speed of the sheet with a being the magnitude of the shrinking or stretching rate. The gas slip velocity at the moving surface is [12]

$$u_{slip}=b_s\lambda\frac{\partial u}{\partial y}-c_s\lambda^2\frac{\partial^2 u}{\partial y^2}+\frac{4\dot{M}}{\alpha\rho\bar{v}}u_w, \tag{7}$$

where

$$b_s=2(3-\alpha f^3)/(3\alpha)-(1-f^2)/K_n, \tag{8}$$

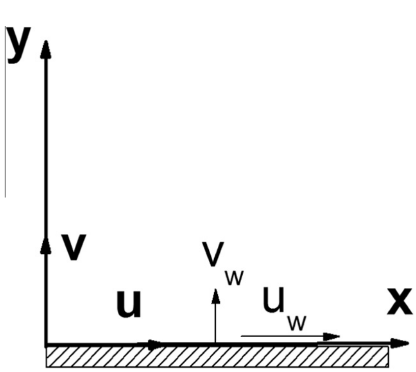

Fig. 1. System setup for viscous gas flow above a shrinking/stretching sheet with mass suction/injection.

and

$$c_s=f^4/4+(1-f^2)/(2K_n^2), \tag{9}$$

are the first and second order slip coefficients due to velocity shearing, λ is the mean free path of gas molecules, K_n is the Knudsen number, $f=\min[1/K_n,1]$ is a dimensionless function, and $\bar{v}$ is the mean molecular speed, respectively. The accommodation coefficient α has a value in the range from 0 to 1 depending on the surface property. The mass flux at wall is $\dot{M}=\rho v_w$, consequently the slip velocity at the moving surface can be written as

$$u_{slip}=b_s\lambda\frac{\partial u}{\partial y}-c_s\lambda^2\frac{\partial^2 u}{\partial y^2}\pm m_1 ax, \tag{10}$$

where

$$m_1=4v_w/(\alpha\bar{v}), \tag{11}$$

is a dimensionless mass-transfer induced slip parameter. The third term on the right-hand-side (RHS) of Eq. (10) has a negative sign for a shrinking sheet and a positive sign for a stretching sheet. When mass transfer is due to evaporation or condensation, m_1 can be further simplified to

$$m_1=\frac{p_{ev}-p}{\alpha p}, \tag{12}$$

where p_{ev} is the equilibrium vapor pressure of the evaporating/condensing gases [12]. The mass transfer induced slip parameter m_1 can be large in a wide range of practical applications. For example when the mass flux is large, or when the surface accommodation coefficient α is small (ranging from 0 to 1). One practical example for m_1 to be large is for the case when the mass transfer is due to evaporation/condensation and the magnitude of the difference between the equilibrium vapor pressure p_{ev} and the vapor pressure p is large compared with the vapor pressure as shown by Eq. (12). In a slightly different geometry setup, i.e., a gas bearing system, we already demonstrated that evaporation/condensation induced slip may have a significant impact on gas flows [12].

We now introduce the stream function and similarity variable

$$\psi(x,y)=f(\eta)x\sqrt{\nu a}, \tag{13}$$

$$\eta=y\sqrt{\frac{a}{\nu}}, \tag{14}$$

so that

$$u(x,y)=axf'(\eta), \tag{15}$$

$$v(x,y)=-\sqrt{\nu a}f(\eta). \tag{16}$$

With the introduction of the stream function the continuity equation (1) is automatically satisfied, and the momentum equations (2) and (3) transform into

$$f'''+ff''-f'^2=0, \tag{17}$$

subject to the following boundary conditions

$$f(0)=-\frac{v_w}{\sqrt{\nu a}}=m_2, \tag{18}$$

$$f'(0)=\pm(1+m_1)+\beta f''(0)+\gamma f'''(0), \tag{19}$$

$$f'(+\infty)=0, \tag{20}$$

where m_2 is the mass transfer parameter, and

$$\beta=b_s\lambda\sqrt{\frac{a}{\nu}}, \tag{21}$$

$$\gamma=-c_s\lambda^2\frac{a}{\nu}, \tag{22}$$