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A general exact analytical solution for anisotropic non-axisymmetric heat conduction in composite cylindrical shells

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ABSTRACT

In this study, a solution for anisotropic conductive heat transfer in a composite cylindrical shell is computed by an exact analytical method. The fibers are considered to be winded around the cylinder at any arbitrary angle. A general linear boundary condition is applied on both circular bases of the cylindrical shell to account for various combinations of thermal conditions. The solution considers the effects of convection of the ambient flow motion and various external radiative heat fluxes. The analytical solution describes the temperature distribution in the axial and circumferential directions. In principle, the heat conduction equation should involve a dual second-order derivation, which precludes solving the equation by the direct application of common exact methods. Therefore, an appropriate canonical mapping is selected as a solution to cancel the dual derivation of temperature in the mapped equations. The separation of variables method is then applied to the mapped equations, which are conducted to obtain an exact form of the solution. Finally, an analytical technique based on the Fourier series concept is proposed to determine unknown coefficients in the final solution. The obtained analytical results correspond well with the solved numerical results. The capability of the current solution is examined by application to a few relevant industrial cases.

1. Introduction

Composite structures have been the subject of considerable research interest due to their many industrial applications. Compared to other raw materials, composites have superior features, such as high ratios of strength and stiffness to density, high corrosion resistance, and plasticity, which are advantageous in myriad industrial applications, including piping, heat exchangers, fluid reservoirs, pressure vessels, aerospace components, and others. Understanding heat conduction in composite materials is important for many applications, such as preventing thermal fracture [5–7], analyzing fiber placement in the production process [8–10], and controlling the directional heat transfer in laminates.

Previous studies have concentrated on the mechanical and thermo-mechanical behaviors of composite materials [1–4], with a few being devoted to conductive heat transfer. Studies of heat conduction in composite materials typically have implemented either numerical or approximated semi-analytical methods, whereas only a few studies have introduced fully analytical solutions.

Demuth et al. [11] investigated the accuracy of the predicted effective thermal conductivity of composite materials, as determined by the finite volume and thermal lattice Boltzmann methods using single relaxation time. They considered two- and three-dimensional heat conduction problems in two-phase domains with a high thermal conductivity ratio, as well as the analytical expression for effective thermal conductivity. Turhan and Tuna [12] extended an approximation continuum theory on elasticity to obtain the solution for heat conduction in infinite composite slabs; however, they did not consider the anisotropic case. Chang et al. [13] transformed the heat conduction differential equation into integral equations by solving Green's functions for three specific problems under the steady and transient states.

Using an analytical technique, Sarkar et al. computed the temperature distribution within a heat-generating cylinder with anisotropic thermal conductivity [14]. The cylinder was subjected to a convective heat transfer coefficient that varied spatially due to fluid flow. Their solution considered the effects of various parameters, including the Reynolds number of the flow and extent of anisotropy. The results of this solution may be useful for the convection-based thermal management of lithium ion cells. Tokovyy and Ma presented an efficient technique for analytically

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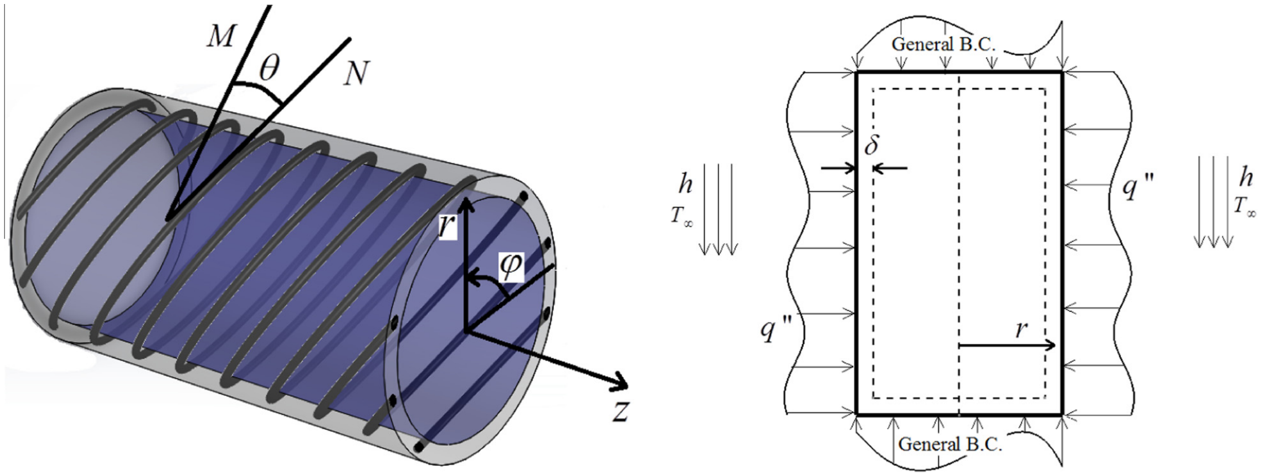

Fig. 1. Directions of fibers and boundary condition definitions in composite cylindrical shell.

Table 1
Specifications of three types of composite materials.

Material number & name	Filler	Matrix	k_{11} (W/mK)	k_{22} (W/mK)
1. Carbon–carbon [49]	Carbon	Carbon	20	1.3
2. DKD–Lexan	Thermal Graph DKD X	Lexan HF 110-11 N	8	0.6
3. Thermocarb–Zytel	Thermocarb CF300	Zytel 110 NC010	1.1	0.4

Table 2
Geometrical parameters of the cylindrical shell.

Radius of the cylinder (r)	1 m
Height of the cylinder (L)	2 m
Thickness (δ)	0.01 m

Table 3
Boundary conditions of case 1.

Ambient temperature (T_∞)	300 K
Convection coefficient (h)	10 W/m^2 K
Heat flux (q'')	1357 W/m^2 [48]
heat generation (u''')	0 W/m^3
$F_{1,2}$ (General B.C)	−27
$u_{1,2}$	1
$v_{1,2}$	0

solving three-dimensional heat conduction and thermal stress problems for an elastic layer [15]. They directly integrated the original heat transport and thermoelasticity equations, reducing them to the governing Volterra integral equations of the second kind. Ma and Chang [16] transformed an anisotropic to an isotropic problem by linear coordinate transformation and obtained an analytical method to determine heat conduction in anisotropic media.

Haji-Sheikh et al. [17] derived a formulation for steady-state conductive heat transfer in multilayer bodies. They reported real eigenvalues in the homogeneous layers, but complex eigenvalues in the orthotropic cases. Extending the analytical model for two-layer bodies developed by Haji-Sheikh et al. [17] and assigning a volumetric heat source to each surface heat flux, Kaisare et al. [18] obtained an analytical solution for the temperature distribution of a first-level package with a nonuniformly powered die. Using an inverse methodology, they solved the equations for various surfaces to calculate the maximum junction temperature for a given multilayer body.

Singh et al. [19] used the finite integral transform method to solve the problem of asymmetric heat conduction in a multilayer annulus with time-dependent boundary conditions. An eigenfunction expansion approach was used to satisfy the periodic boundary condition in the angular direction. In this method, the original partial differential equation (PDE) was transformed to an ordinary differential equation (ODE), the solution of which was inverted to obtain the temperature distribution in each layer. Miller and Weaver [20] presented an analytical solution for heat conduction in multilayer bodies subjected to convective and radiative boundary conditions. Singh et al. [21] addressed conductive heat transfer in the polar coordinate system for a multilayer medium. They developed an analytical two-dimensional solution by applying the separation of variables method. Using the appropriate Green's functions, Huang and Chang [22] obtained an analytical solution for heat conduction in multilayer composite materials under steady-state, periodic, and unsteady conditions.

Miller and Weaver [23] used finite element and analytical approaches to determine the transient temperature distribution through an air-filled box structure with carbon fiber-reinforced plastic skins. They developed a novel method for including natural convection within a cavity by using ABAQUS, and proposed integral transform-based analytical models to describe the temperature distribution through a carbon fiber-reinforced plastic skin under combined thermal loading conditions. Lu et al. [24] solved the transient temperature distribution in a multidimensional composite circular cylinder using a separation of variables-based analytical method. They considered time-dependent temperature variation to be a boundary condition. Similarly, in another work, Lu et al. [25] solved the problem of transient heat conduction in a one-dimensional hollow composite cylinder with a time-dependent boundary temperature. Applying Laplace transformation, they presented a closed form of a newly derived approximate analytical solution.

Blanc and Touratier [26] developed an analytical model for heat conduction in multilayered structures based on an equivalent