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General theory of instantaneous power for multi-phase systems with distortion, unbalance and direct current components

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ABSTRACT

Active instantaneous currents are generally defined as those compensated supply-wire currents that deliver a given instantaneous power with minimum line losses, without a change in voltage. Since the concept was introduced 60 years ago, many theories have been proposed to enable the calculation of those optimum supply currents, for various conditions of the supply system. This paper shows how these optimal wire currents can be obtained with a single general formula applicable to all supply systems. The solution depends on the number of wires considered, their resistances, which need not be equal, and their respective voltages measured from a common reference. The formula is derived through the properties of linear algebra in vector space, and is a direct consequence of Kirchhoff's current law and the law of conservation of energy. All the existing theories can be identified as particular cases of the general formula and most can be grouped into three common categories.

1. Introduction

Common power theory concepts, such as apparent power and reactive power developed in the first part of the 20th century, have been of great benefit in modelling symmetrical, sinusoidal systems in electrical engineering. However, with the increasing presence of nonlinear loads, conventional power theory is sometimes inadequate or incorrect when applied to systems with unbalance, distortion or direct current components. Various approaches have been proposed to meet these conditions. Standards and definitions have been adopted and revised [1,2], but inconsistencies still have technical and financial consequences for power systems design, metering, and quality of supply regulation.

Two approaches have dominated power theory development in the last 60 years: instantaneous power and average power. So-called instantaneous collective power theory was introduced by Buchholtz [3] and taken up by Depenbrock [4], but both were initially not widely known. Another instantaneous power theory was popularised by Akagi et al. in 1983 [5] and spurred many publications and debates, in which authors reformulated Akagi's theory, offered other instantaneous theories, or extended the instantaneous power theories into the average power domain. Other researchers based their theory on average power and the frequency domain, and have even questioned whether instantaneous power theories can be extended accurately to the average power domain [6]. Generally, all these power theories decompose the load currents into a component contributing active ("useful") current and one or more non-active[1] ("useless") current components that can be provided by a local compensator, thereby reducing the transmission losses – which is the main justification for the continuing interest in power theory.

Building on and extending initial work by Malengret and Gaunt [7] and Malengret [8] on 3- and 4-wire systems, this paper reviews the existing theories, presents a general theory of instantaneous power for systems with any number of wires of any resistance, and shows that all the other instantaneous power theories are particular cases of the general theory. Using the same mathematical approach, a companion paper [9] extends this general theory of instantaneous power to the domain of average power for systems with any number of wires of any resistance, formulating an internally consistent general power theory valid for instantaneous and average power. A further companion paper [10] describes the implementation of the general power theory in practical measurement circuits and discusses some implications for power systems.

2. Early definitions of power for single-phase systems

Active power P supplied by a periodical voltage source to a single-phase load is the average power over an observation time T, the duration of one cycle or an integer number of cycles, given

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[1] Due to controversy over the term "instantaneous reactive power", most publications now refer to non-active current or "useless current" and non-active power.

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by $P = 1/T\int_T u(t)i(t)dt = 1/T\int_T p(t)dt$ where $p(t) = u(t)i(t)$ and can be regarded as the time rate of energy transfer or consumption, $u(t)$ is the voltage between the two wires, and $i(t)$ the current in them.

When the voltage and current are sinusoidal, it can be shown for a single phase supply that the average power P is the product of the rms value of the voltage and current, U and I respectively, multiplied by the cosine of the angle between the voltage and the current, which is also known as the power factor (p.f.), so that $P = UI\cos\varphi$. When the waveforms are not sinusoidal and periodic, they can be expressed as a Fourier series and it can be shown from Fourier analysis that the average power P is the sum of the individual harmonic powers.

There is no controversy about the definition of average power; also apparent power in single phase systems has a unique and invariant value for any particular current and voltage waveform. However apparent power and 'reactive' powers are not based on a single well defined physical phenomenon, only on similar models.

For sinusoidal voltages and currents, reactive power (also known as non-active power, imaginary power or "useless" power as it does not contribute to the average power used by a load) is defined as $Q = UI\sin\varphi$, and apparent power as $S = UI$, with a Pythagorean relationship $S^2 = P^2 + Q^2$ in which Q can be assigned a positive or negative value by convention. In the case of non-sinusoidal single phase waveforms, the definition for apparent power S is also $S = UI$ where U and I are the rms values of $u(t)$ and $i(t)$ respectively, but the non-active power has been defined in various ways by different researchers.

An early idea of reactive power Q in non-sinusoidal single-phase systems was introduced by Fryze in 1932 [11] and recommended 50 years later by the International Electrotechnical Commission [1]. According to Fryze, if $u(t)$ is the voltage of a single-phase system, then the source current $i(t)$ can be decomposed in the time domain into components $i_a(t)$ and $i_b(t)$. $i_a(t)$ is defined as $i_a(t) = (P/U^2)$ $u(t) = Gu(t)$ where P is the average power supplied to a load during an interval T, U is the rms value of the voltage applied to the load, and conductance $G = P/U^2$. The remaining current $i_b = i(t) - i_a(t)$ makes no contribution to the average power. Using $S = UI$, and defining $Q = UI_b$, it can be shown that $S^2 = P^2 + Q^2$, thereby defining Q without the use of Fourier series. Q can be regarded as representing a measure of the under-utilisation of a single-phase system, but it has no physical interpretation equivalent to a sinusoidal single phase system.

Thus, Fryze introduced the concept that $i_a(t)$ is proportional and in phase with the voltage, and is optimum in the sense that it can be shown mathematically that no other current would result in lower line losses while delivering the same power at the same voltage. Further, the conductance G is a constant term based on average power, although at the time there was no distinction between average and instantaneous power that has been taken up by subsequent researchers in a similar form as an instantaneous term with a similar meaning.

3. Extending single phase theory to instantaneous power in multi-phase systems

Although three phase, 3- and 4-wire systems were in common use, it was nearly 20 years before Fryze's single phase definition of non-active power was extended to poly-phase systems, and it is useful to review some of the contributions made by the instantaneous power theories that followed.

3.1. Buchholtz' instantaneous power

Buchholtz appears to have been the first to extend Fryze's approach to polyphase systems. Based on Fryze's concept for active currents in single phase systems, Buchholtz [3] formally introduced the concept of the instantaneous collective values of current and voltages. In a system with M phases, he defined the instantaneous active current of the νth wire, $i_{\nu p} = g_p u_\nu$, as proportional to the conductance g_p and a voltage u_ν, which can be calculated at any instant, from the instantaneous values of voltages and currents. The neutral wire was not defined as the reference for the voltage measurements, and u_ν is calculated from the voltage differences between the wires, so is independent of the voltage reference chosen. The theory can be applied to any number of wires, but it is not clear that it provides an optimum solution (in the sense that the transmission losses will be minimum after compensation), as no mathematical proof is given.

Buchholtz' instantaneous power theory was not well known, apart from Depenbrock who included it in 1962 in what he called the FBD power theory [4]. Buchholtz' and Depenbrock's work was published in German initially and was only appreciated more widely when it appeared in English publications from 1993 [12–15].

3.2. Akagi's instantaneous p–q theory

Apparently unaware of Buchholtz' instantaneous current theory in poly-phase systems, Akagi et al. [5,16] proposed a time domain theory for three-phase systems called the p–q instantaneous power theory, introducing a new electrical quantity q called instantaneous reactive power.

Akagi began by defining voltage and current vectors $\boldsymbol{E}_{abc}$ and $\boldsymbol{I}_{abc}$ and transformed them to the 0–α–β reference frame:

$$\boldsymbol{E}_{0\alpha\beta} = \{e_0, e_\alpha, e_\beta\}^T = A\boldsymbol{E}_{abc} \quad \text{and} \quad \boldsymbol{I}_{0\alpha\beta} = \{i_0, i_\alpha, i_\beta\}^T = A\boldsymbol{I}_{abc} \tag{1}$$

where

$$A = \sqrt{2/3}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & \sqrt{3}/2 \end{bmatrix}$$

The instantaneous power is $p = e_a i_a + e_b i_b + e_c i_c$, and the original theory defines two instantaneous real power components p_0 and $p_{\alpha\beta}$ and one instantaneous imaginary power $q_{\alpha\beta}$, calculated as:

$$\begin{bmatrix} p_0 \\ p_{\alpha\beta} \\ q_{\alpha\beta} \end{bmatrix} = \begin{bmatrix} e_0 & 0 & 0 \\ 1 & e_\alpha & e_\beta \\ 0 & -e_\beta & e_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} \tag{2}$$

This leads to the decomposition of the currents into three 0–α–β components in each phase, from which the compensating currents can be found and transformed back into the abc reference frame using the inverse matrix of A. An active filter requiring no instantaneous real power can supply compensating currents that are the non-active components of the load current. The compensated supply current still supplies the real power, with reduced line losses.

The p–q theory attracted interest because reactive power was normally attributed only to sinusoidal waveforms at that time. The transform's appearance was familiar to many electrical engineers, and was of immediate benefit in that it enabled calculation in real time of compensating currents for active filters without energy storage. The theory was applied to 3-phase, 3- and 4-wire systems, and gives the same result as Buchholtz' approach with $m = 3$. However, in 4-wire systems Akagi's resultant neutral current was zero, different from Buchholtz' approach with $m = 4$, and this draws attention to the lack of a mathematical justification that either solution provided optimum compensation.

3.3. Willems' vector approach

Willems [17,18] proposed a more direct formula than Akagi's to calculate instantaneous current components. His approach is that