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Numerical simulations of Oldroyd 8-constant fluid flow and heat transfer in a curved channel

N. Ali [a], K. Javid [a,*], M. Sajid [b], A. Zaman [a], T. Hayat [c,d]

[a] Department of Mathematics and Statistics, International Islamic University, Islamabad 44000, Pakistan
[b] Theoretical Physics Division, PINSTECH, P.O. Nilore, Islamabad 44000, Pakistan
[c] Department of Mathematics, Quaid-I-Azam University, Islamabad 44000, Pakistan
[d] Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21598, Saudi Arabia

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ABSTRACT

A study is carried out to investigate the heat transfer characteristics in peristaltic flow of Oldroyd 8-constant in a curved channel when inertial and streamline-curvature effects are negligible. The solution of resulting nonlinear governing equations is obtained using finite difference technique (FDM) combined with an iterative scheme. The impacts of physical parameters on the flow and heat transfer characteristics is investigated in detail. Particular attention is given to explain the pumping and trapping phenomena in detail. A comparative study between curved and straight channels is also made. It is found that, the rate of heat transfer increases with increasing the curvature of the channel. The current two-dimensional analysis is applicable in bio-fluid mechanics, industrial fluid mechanics, and some of the engineering fields.

1. Introduction

Numerous biological conduits, including the digestive system [1], ureter [2], male reproductive tract, fallopian tube, bile duct, and oesophagus transport their fluid substances by peristalsis – the driving force of internal fluid by propagating waves of muscular contraction in the surrounding tube wall. Generally this pumping phenomena fall by from the region of lower pressure to the region of higher pressure. Importance of peristalsis is quite prevalent in the industrial and physiological applications. This phenomenon is extensively studied in the past few decades. The specialized literature for peristalsis mechanism is reviewed in the following paragraph.

The theoretical analysis regarding the above mechanism was developed by Shapiro et al. [3] and Fung and Yin [4] for Newtonian fluid case. The effects of inertia and streamline curvature wave reported by Jaffrin [5]. Takabataka et al. [6] and Brown and Hung [7] numerically investigated two-dimensional peristaltic transport without using long wavelength and low Reynolds number assumptions. The first study regarding peristaltic flow of non-Newtonian fluids was investigated by Raju and Devanathan [8] using Power-law fluid. Srivastava [9] employed couple stress theory to investigate the peristaltic motion. Peristaltic flow of second and third order fluids was analyzed by Siddiqui et al. [10] and Siddiqui and Schwarz [11]. Hayat et al. carried out the analysis of peristaltic motion for different non-Newtonian fluids including third order fluid [12], Johnson–Segalman fluid [13], Oldroyd-B fluid [14] etc. Ali et al. theoretical analyzed peristaltic motion of Oldroyd 4-constant fluid [15], Oldroyd 8-constant fluid [16], Giesekus fluid [17]. All the above mentioned studies do not take into account the effects of curvature on the peristaltic motion. The first study incorporating the effects of curvature in peristaltic motion was carried out by Sato et al. [18]. Later attempts involving non-Newtonian effects in peristalsis through curved channel were made by Ali et al. [19,20], Hayat et al. [21,22], Hina et al. [23–25], Abbasi et al. [26], Narla et al. [27], Kalantari et al. [28] etc. However, very little is said about heat transfer in peristaltic flow through curved channel despite the extensive literature on heat transfer in peristalsis through straight geometries. The study conducted by Vajravelu et al. [29] for Newtonian fluid spearheaded further developments in modeling of peristalsis with heat transfer. Later attempts in this direction to include mass transfer, non-Newtonian effects, wall geometry etc. were presented by Hayat et al. [30], Srinivas and Kothandpani [31], Tripathi and Beg [32] to name only a few.

The first rigorous attempt to model heat transfer in peristaltic flow of Newtonian fluid in curved channel was presented by Ali et al. [33]. They showed that temperature of fluid inside the

* Corresponding author. Tel.: +92 051 9019756.
E-mail address: khurram_javid1985@yahoo.com (K. Javid).

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Nomenclature

a_1	half width of curved channel
b_1	amplitude of the peristaltic wave
Br	the Brinkman number
c	the wave speed
c_p	specific heat at constant pressure
$\bar{\boldsymbol{I}}$	the identity tensor
k	dimensionless curvature of the channel
k^*	thermal conductivity
O	center of curvature of the channel
$\bar{p}$	pressure in wave frame
$\bar{P}$	pressure in fixed frame
q	time-averaged flow rate in wave frame
$\bar{r}$	radial coordinate in wave frame
$\bar{R}$	radial coordinate in fixed frame
R^*	dimensional radius of curvature
Re	the Reynolds number
$\bar{\boldsymbol{S}}$	the extra stress tensor
$\bar{t}$	the time
$\bar{\boldsymbol{T}}$	temperature in fixed frame
T_0	temperature at lower wall
T_1	temperature at upper wall
$\bar{u_1}$	the axial velocity component in wave frame
$\bar{u_2}$	the radial velocity component in wave frame
$\bar{U_1}$	the axial velocity component in fixed frame
$\bar{U_2}$	the radial velocity component in fixed frame
$\bar{x}$	axial coordinate in wave frame
$\bar{X}$	axial coordinate in fixed frame
z	heat transfer coefficient
(X', Y')	Cartesian coordinates fixed at O

Greek symbols

η	the dimensionless radial distance in wave frame
β	coefficient of thermal expansion
$\bar{\tau}$	the Cauchy stress tensor
Φ	dissipation function
$\bar{\lambda}_i\,(i = 1-7)$	the material constants
λ^*	the wavelength
ρ	the fluid density
δ	the wave number
θ	the dimensionless temperature in wave frame
Θ	time-averaged flow rate in fixed frame
$\bar{A_1}$	the first Rivlin–Ericksen tensor
ϕ	amplitude ratio
ψ	stream function

channel increases by increasing the curvature of the channel. Later studies in this direction were carried out by Hayat et al. [34], Nadeem et al. [35], and Hina et al. [36]. Apart of above mentioned studies no other attempts is available in the literature dealing with flow and heat transfer analysis of a non-Newtonian fluid in a curved channel. It is observed that numerous industrial processes require the deep knowledge of heat transfer and the corresponding thermal coefficients. These include condensation, crystallization, evaporation and other boiling operations. For instance, the production of orange juice concentrate, concentrated H_2SO_4, and distilled water is based on the evaporation technique. In physiology evaporation technique is used to investigate the thermal properties of tissues/cells. Moreover, thermodynamical aspects of blood may influence the processes like oxygenation and hemodialysis when blood is drawn out of the body. The application of heat energy in techniques like laser therapy and cryosurgery for treatment of malicious canner cells have also stimulated much of interest in study of thermal modeling in tissue. The interaction of peristalsis with heat transfer have also demonstrated several interesting aspects of bolus dynamics in gastro-intestinal tract since the thermal properties of fluid may affect the bolus transport. The above mentioned facts motived us to further venture in the regime of flow and heat transfer in curved channel. In particular, we propose to carry out an analysis of heat transfer in peristaltic flow of Oldroyd 8-constant fluid in a curved channel.

The problem is modeled using fundamental laws of mass, momentum and energy concentration under long wavelength and low Reynolds number assumption. The modeled equations are simulated using a robust implicit finite difference technique. Hence, the effects of various emerging parameters on flow and heat transfer characteristics are reported.

2. Mathematical formulation

We consider incompressible flow of an Oldroyd 8-constant fluid in a curved channel of width $2a_1$. The channel is coiled in a circle with center O and radius R^*. The flow in the channel is generated by sinusoidal waves of small amplitude b_1 traveling with speed c along the channel walls. The upper and lower walls of the channel are maintained at constant temperature T_1 and T_0, respectively. The schematic diagram of the flow geometry is given in Fig. 1. A curvilinear coordinate system $(\bar{R}, \bar{X})$ is used to analyze the flow in which $\bar{R}$ is along the radial direction and $\bar{X}$ is along the axial direction.

This curvilinear coordinate system is related with the Cartesian coordinate system (X', Y') fixed at O through the following transformations.

$$\left.\begin{aligned} X' &= (R^* + \bar{R})\cos\left(\frac{\bar{X}}{R^*}\right), \quad \text{(a)} \\ Y' &= (R^* + \bar{R})\sin\left(\frac{\bar{X}}{R^*}\right). \quad \text{(b)} \end{aligned}\right\} \tag{1}$$

Now the equation of the upper wall in (X', Y') system is

$$X'^2 + Y'^2 = \left((R^* + a_1) + b_1\cos\left(\frac{2\pi}{\lambda^*}\left(R^*\tan^{-1}\left(\frac{Y'}{X'}\right) - c\bar{t}\right)\right)\right)^2. \tag{2}$$

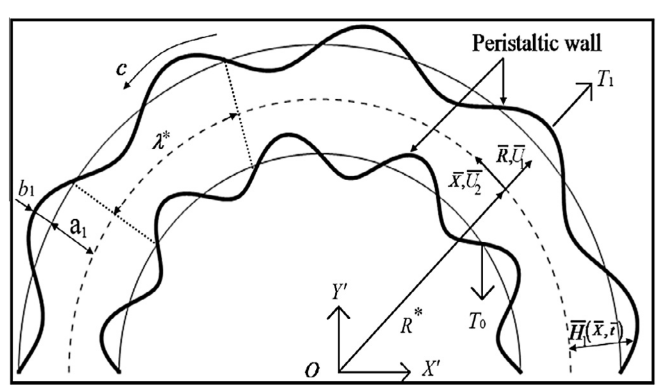

Fig. 1. Schematic diagram of the curved moving boundary problem.