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Lagrangian investigations of vortex dynamics in time-dependent cloud cavitating flows

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ABSTRACT

Lagrangian investigations of vortex dynamics, including Lagrangian Coherent Structures (LCS) and particle trajectory, are conducted to highlight the mechanisms of cloud cavitating flows around a Clark-Y hydrofoil. Numerical simulations are performed using a transport equation-based cavitation model and the large eddy simulation (LES) approach. Good agreements are observed between numerical predictions and experimental measurements, including time-averaged turbulence statistics, velocity, vorticity profiles and the periods of unsteady shedding process of the vortex structures near the trailing edge. Besides, present numerical predictions are capable of capturing the unsteadiness of cloud cavitation, including the initiation, growth toward the trailing edge and subsequent shedding of cavities. Based on the Lagrangian analysis of vortex dynamics in non-cavitating flows, two LCSs, namely LE-LCS and TE-LCS, are defined. In cloud cavitating flows, distributions of the two LCSs in different cavitation developing stages illustrate different behaviors of vortex structures. (a) In the attached sheet cavity growing stage, the LE-LCS extends to the trailing edge, which implies the expansion of the attached sheet cavity, and the TE-LCS rolls up and extends downstream, which implies the detachment of cloud cavity. In addition, particle tracers indicate that the Leading edge vortex (LEV) is enhanced by the attached sheet cavity, and there is no direct interaction between attached cavity's expansion and cloud cavity's shedding. (b) In the re-entrant jet developing stage, the LE-LCS and TE-LCS connect together near the middle of the hydrofoil, which implies that two vortex structures mix together inside of the stable attached cavity. Particle tracers clearly show the re-entrant jet flow and the unsteadiness of the vortex structures inside of the stable attached cavity. Furthermore, trapped particles tracers indicate the semi-Vortex Street in the wake, which is induced by the resistance effect from the stable cavity on the partial shedding of the LEV. (c) In the cloud cavity shedding stage, no connection between the LE-LCS and TE-LCS can be observed near the middle part of the hydrofoil, which implies the break-up of vortex structures inside the attached cavity. Meanwhile, particle tracers show the breakup of vortex structure inside the attached cavity, as well as the shedding process of the rear part, which is enhanced by the detached cloud cavity.

1. Introduction

Cavitation is a dynamic phase-change phenomenon that occurs in liquids when the static pressure drops below the vapor pressure of liquid [1,2]. It is well known that the unsteady cavitation in turbo-machinery and marine control surfaces will lead to problems such as material damage, vibration, noise and reduced efficiency [3,4]. A particularly important form of cavitation is that on the suction side of lifting surfaces. At typical angles of attack, as the local pressure reduces, lifting surfaces cavity structures change from localized, instantaneous pressure drops typical of inception cavitation [5], to sustained, time-dependent cavities typical of sheet, cloud, and super-cavitation [6–9].

Highly vortical fluid motion is often observed downstream of the attached cavitation. This motion is caused by vorticity shedding into the flow field slightly downstream of the cavity. Such vortex cavitation generates a large cavitation cloud under certain conditions [10]. It has been proven that the cloud cavitation is a large-scale vortex with many small cavitation bubbles [11,12]. In addition, various studies have demonstrated the strong correlation between cavitation and vortex structures. Gopalan and Katz [13] observed that the collapse of the vapor structure is a primary mechanism of vorticity production and leads to the generation of hairpin vortices in the downstream region. Iyer and Ceccio [14] and Laberteaux et al. [15] found that the stream-wise velocity

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fluctuations increased with increased cavitation, which confirms that the collapse of the vapor cavities is a source of vorticity generation. Additionally, Dittakavi et al. [16] and Huang et al. [17,18] discussed the influence of cavitation on different terms of vorticity transport equation, and found that the periodic formation, breakup, shedding, and collapse of the sheet/cloud cavities, and the associated baroclinic and viscoclinic torques, are important mechanisms for vorticity production and modification. Ji et al. [19–22] also analyzed the three-dimensional cavity structures around a twisted hydrofoil, and the detail analysis using the vorticity transport equation showed the cavitation accelerates the vortex stretching and dilatation and increase the baroclinic torque as the major source of vorticity generation.

Strong correlations between cavitation and vortex structures in cavitating flows highlight the importance of better understanding of unsteady vortex behaviors. During these years, many popular and powerful Lagrangian techniques have been developed to highlight unsteady vortex structures [23–25]. Among these methods, Lagrangian Coherent Structures (LCS) are widely used for many outstanding advantages [26–28]. Lipinski and Mohseni [29], Shadden et al. [30] and Franco et al. [31] used the LCS to exam unsteady vortex flows produced by halobios, results of which revealed a well-defined, unsteady recirculation zone that is not apparent in the corresponding classical Eulerian fields. Besides, Green et al.'s research [32] on turbulent flows demonstrated that the LCS method could define time-dependent vortex structure boundaries without relying on a pre-selected threshold, and presented greater visible details without the requirement of velocity derivatives. Although Tang et al. [33] utilized the LCS method to flow dynamics and underlying physics of unsteady turbulent cavitating flows, no further mechanism about vortex–cavitation interrelations was discussed.

The objective of this paper is to investigate the vortex dynamics and vortex–cavitation interactions in unsteady cloud cavitating flows. The aims are to (1) utilize Lagrangian based methods onto cloud cavitating flows, and assess their ability to analyze cloud cavitating vortex structures, (2) improve the understanding of unsteady vortex structure behaviors in cloud cavitating flows, (3) provide an insight of the vortex–cavitation interrelations in cloud cavitating flows. In the present paper, summary of the Lagrangian methods are presented in Section 2. Numerical setups and validations are shown in Sections 3. In Section 4, Lagrangian investigations of unsteady vortex behavior in non-cavitating flows are firstly presented, followed by detailed analysis of time evolutions of unsteady cavitation, vortex structures and the vortex–cavitation interrelations in cloud cavitating flows.

2. Lagrangian coherent structures

Haller and Yuan [34] developed the LCS approach from a Lagrangian perspective by considering the fluid as a dynamical system of fluid particles rather than a continuum. A clearer presentation of the theory and computation details of the LCS has been established by Shadden et al. [30], enabling this method to become more useful to evaluate the flow structures in a wide range of scope. Considering two points x_0 and $\boldsymbol{x}_0 + \delta\boldsymbol{x}_0$, each of which will generate a trajectory in the space. Use one of the trajectories as a reference, the divergence between the two trajectories can be written as a function of the time and the initial location with the form of $\delta\boldsymbol{x}(\boldsymbol{x}_0, t)$. The mean exponential rate of separation of two close trajectories can be computed by using the following formula,

$$\sigma = \lim_{\substack{t\to\infty \\ |\delta\boldsymbol{x}_0|\to 0}} \frac{1}{t} \ln \frac{|\delta\boldsymbol{x}(\boldsymbol{x}_0, t)|}{|\delta\boldsymbol{x}_0|}, \tag{1}$$

σ is the Lyapunov Exponent. The Lyapunov Exponent of a dynamic system is a quantify that characterizes the rate of separation of infinitesimally close trajectories. The finite time version of the Cauchy-Green deformation tensor, Δ, at the given point $\boldsymbol{x}_0$ is defined as,

$$\Delta_{t_0}^{T_{\mathrm{L\_E}}}(\boldsymbol{x}_0) = \left(\frac{\partial\boldsymbol{x}(t_0+T_{\mathrm{L\_E}};\ t_0,\ \boldsymbol{x}_0)}{\partial\boldsymbol{x}_0}\right)^T \frac{\partial\boldsymbol{x}(t_0+T_{\mathrm{L\_E}};\ t_0,\ \boldsymbol{x}_0)}{\partial\boldsymbol{x}_0}, \tag{2}$$

where $()^T$ is the transpose of the deformation gradient tensor. The maximum eigenvalue of $\Delta_{t_0}^{T_{\mathrm{L\_E}}}(\boldsymbol{x}_0)$ is defined as $\lambda_{\max}\left(\Delta_{t_0}^{T_{\mathrm{L\_E}}}(\boldsymbol{x}_0)\right)$. It represents the maximum stretching, and the corresponding eigenvalue provides the direction and vector, which $\delta\boldsymbol{x}_0$ will align to. Then the largest finite-time Lyapunov Exponent with a finite integration time $T_{\mathrm{L\_E}}$ is defined as:

$$\sigma_{t_0}^{T_{\mathrm{L\_E}}}(\boldsymbol{x}_0) = \frac{1}{|T_{\mathrm{L\_E}}|} \ln \sqrt{\lambda_{\max}\left(\Delta_{t_0}^{T_{\mathrm{L\_E}}}(\boldsymbol{x}_0)\right)}. \tag{3}$$

The Finite-time Lyapunov Exponent (FTLE) represents the maximum stretching rate for infinitesimal close particles. Ridges in the FTLE field are named as LCS. It is shown that the LCS can define structure boundaries without relying on a preselected threshold and present greater visible details without the requirement of velocity derivatives. Computation details can be read in Ref. [34].

3. Numerical setup and description

The turbulent cavitating flows are solved using the unsteady Navier–Stokes equations coupled with the large eddy simulation (LES) and a mass transfer cavitation model.

3.1. Governing equations and large-eddy simulation approach

In the mixture model of the vapor/liquid two-phase flow, the multiphase components are assumed to have the same velocity and pressure. The governing equations consist of the mass and momentum conservation equations:

$$\frac{\partial\rho_m}{\partial t} + \frac{\partial(\rho_m u_j)}{\partial x_j} = 0, \tag{4}$$

$$\frac{\partial(\rho_m u_i)}{\partial t} + \frac{\partial(\rho_m u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu_m \frac{\partial u_i}{\partial x_j}\right), \tag{5}$$

$$\frac{\partial\rho_l\alpha_l}{\partial t} + \frac{\partial(\rho_l\alpha_l u_j)}{\partial x_j} = \dot{m}^+ + \dot{m}^-. \tag{6}$$

Here u is the velocity, p is the pressure, ρ_l is the liquid density, ρ_v is the vapor density, α_v is the vapor fraction, α_l is the liquid fraction. The source term $\dot{m}^+$, and the sink term $\dot{m}^-$ represent the condensation and evaporation rates, respectively. The mixture density ρ_m and dynamic viscosity μ_m are defined as:

$$\rho_m = \rho_l\alpha_l + \rho_v\alpha_v, \tag{7}$$

$$\mu_m = \mu_l\alpha_l + \mu_v\alpha_v. \tag{8}$$

Applying a Favre-filtering operation to Eqs. (4) and (5) gives the LES equations,

$$\frac{\partial\rho_m}{\partial t} + \frac{\partial(\rho_m \bar{u}_i)}{\partial x_j} = 0, \tag{9}$$

$$\frac{\partial(\rho_m \bar{u}_i)}{\partial t} + \frac{\partial(\rho_m \bar{u}_i \bar{u}_j)}{\partial x_j} = -\frac{\partial\bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu_m \frac{\partial\bar{u}_i}{\partial x_j}\right) - \frac{\partial(\rho_m\tau_{ij})}{\partial x_j}, \tag{10}$$

where over bar denotes a filtered quantity. By comparing Eq. (10) with Eq. (5), an extra nonlinear term, τ_{ij}, which is called the sub-grid scale (SGS) stress, is expressed as:

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i\bar{u}_j. \tag{11}$$