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Magnetohydrodynamic counter-rotating flow in a cylindrical cavity

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ABSTRACT

Three-dimensional steady combined free and forced convective magnetohydrodynamic (MHD) flow are simulated in a cylindrical cavity filled with a liquid metal and submitted to a vertical temperature gradient and an axial magnetic field. The forcing corresponds to a swirling flow produced by counter-rotation of the top and bottom disks. The governing Navier–Stokes, energy, and potential equations along with appropriate boundary conditions are solved by using the finite-volume method. Comparisons with previous results were performed and found to be in excellent agreement. The effects of magnetic field on flow and temperature fields are analysed. When the Reynolds number is increased, the axisymmetric basic state loses stability and different complex flow appear. Axisymmetric ($m = 0$) and asymmetric $m = 1$ and $m = 2$ azimuthal modes are observed. Azimuthal mode $m = 3$ are found when the Hartmann number, Ha is sufficiently large. In the mixed convection case the $m = 1$ becomes the dominant mode in place of $m = 0$. The stability diagram (Re_{cr}–Ha) corresponding to the transition from axisymmetric to non-axisymmetric flow by increasing values of the Richardson number is obtained.

1. Introduction

Flows between two rotating disks were the subject of many studies. These flows, first studied by Batchelor [1], were given the name of Von Kármán swirling flows by Zandbergen and Dijkstra [2], and occurred frequently in geophysical and industrial applications. There are various studies on Von Kármán flows [3–5]. The motion of a viscous fluid contained in a closed cylinder, with a rotating disk lid which recirculates the flow inside the container poses an attractive example of confined swirling flow and provides very well controlled conditions, particularly so for numerical studies. The major characteristics of the flow are known to be determined by two dimensionless parameters: The aspect ratio ($\gamma = H/R$) and the Reynolds number ($Re = \Omega R^2/\nu$), where H and R are the height and the radius of the cylinder, respectively, Ω is the angular velocity of the top and bottom disks, and ν is the kinematic viscosity of the fluid.

When a magnetic field is applied to a flow of an electrically conducting fluid, complex induction mechanisms occur and induced currents and magnetic field that are generated [6]. In the case of crystal growth, for example, magnetic fields are used to suppress the convective motion induced by the arising strong fluxes in order to control the flow in the melt, and consequently the crystal quality [7]. Except the industrial processing applications, where the flows in cylindrical containers are important, another case with strong theoretical interest is the flow of liquid metals in fusion blankets [8]. Several previous studies considered mainly electromagnetic damping of the flow and its effect on the heat/mass transfer [9]. To control the flow physics, Ben Hadid et al. [10] employed a spectral numerical method to solve the equations of Navier–Stokes of the oscillatory three-dimensional flow of a conducting molten metal confined in a cylindrical cavity. Thus, Ben Hadid et al. [10] examined the temporal signals, the properties of symmetry and the assessments of energy characterising oscillatory flow, and the damping of the flow oscillations by a vertical magnetic field until the stabilization of this flow. Bessaïh et al. [11] carried out a numerical study on rotating MHD steady laminar flow of a liquid metal contained in a cylindrical enclosure, having an aspect ratio equal to 1 and subjected to a vertical external magnetic field. They showed that the primary flow could be controlled by a good choice of electric conductivity of the enclosure walls in question. Bessaïh et al. [12] carried out a numerical and analytical combined study of the MHD flow. They showed a strong dependence of the flow and heat transfer structures with the magnetic field and the electric conductivity of the walls, constituting the cylindrical enclosure. Kharicha et al. [13] carried out a numerical study of a steady laminar MHD flow driven by a rotating disk at the top of a cylindrical cavity filled with a liquid metal. They investigated the effects of the magnetic field, the fluid, the wall electrical conductivities, and the

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Nomenclature

B	magnitude of the external magnetic field (Tesla)
g	gravity (m/s^2)
H	height of the cylinder (m)
Ha	Hartmann number = $BR\sqrt{\sigma/\rho\nu}$
J	dimensionless current density
Nu	local Nusselt number
P	dimensionless pressure
Pr	Prandtl number = ν/α
R	radius of the cylinder (m)
r, θ, z	dimensionless spatial coordinates
Re	Reynolds number = $\Omega R^2/\nu$
Re_m	magnetic Reynolds number = $\mu_0\sigma\Omega R$
Ra	Rayleigh number = $\beta(T_h - T_c)gR^3/\nu\cdot\alpha$
Ri	Richardson number = $Ra/(Pr\cdot Re^2)$
T	temperature (K)
u, v, w	dimensionless radial, axial, azimuthal velocity components
U	dimensionless velocity vector

Greek symbols

α	thermal diffusivity of the fluid (m^2/s)
β	thermal expansion coefficient (1/K)
γ	aspect ratio = H/R
Θ	dimensionless temperature = $(T - T_c)/(T_h - T_c)$
ν	kinematic viscosity of the fluid (m^2/s)
ρ	density of the fluid (kg/m^3)
σ	electric conductivity (Ω/m)
Φ	dimensionless electric potential
Ω	angular velocity (rad/s)
τ	dimensionless time

Subscripts

cr	critical value
c	cold
h	hot

wall thickness. The effect of an externally imposed magnetic field on the axisymmetry-breaking instability of an axisymmetric convective flow, associated with crystal growth from the bulk of the melt, was studied by Gelfgat et al. [14]. It was shown that at relatively small values of Ha, the axisymmetric flow tends to be oscillatory unstable. After, the magnitude of the magnetic field (Ha) exceeds a certain value, the instability switches to a steady bifurcation caused by the Rayleigh–Bénard mechanism. Kakarantzas et al. [15] performed a series of numerical simulations, in order to study liquid metal MHD natural convection in a vertical cylindrical container. The results of this investigation show that the increase of Rayleigh number promotes heat transfer by convection, while the increase of Hartmann number favours heat conduction and vertical magnetic field reduces the Nusselt number more than the horizontal. Recently, Bessaïh et al. [16] studied the MHD stability of an axisymmetric rotating flow in a cylindrical enclosure filled with a liquid metal. They confirmed that the external magnetic field can be used for control and stabilization of the fluid motion and heat transfer. Mahfoud and Bessaïh [17] studied numerically the problem of mixed convection under axial magnetic field. Stability diagrams were established according to the numerical results of this investigation. It was found that the flow between co-rotating end disks is very different from the flow between counter-rotating end disks. The previous works [16,17] investigate numerically the determination of hydrodynamic and thermal instabilities of an axisymmetric rotating flow. In Ref. [18], a numerical simulations were performed in order to study the effects of thermal convection and of a constant axial magnetic field on a von Kármán flow (the objective is to determine the critical Rayleigh number for transition at fixed Reynolds number, $Re = 300$). In the present paper, we continue our previous study [17] with attention has turned to the three-dimensional symmetry breaking of the basic state.

We have studied the three-dimensional combined free and forced convective magnetohydrodynamic flow in a cylindrical cavity having an aspect ratio ($\gamma = H/R = 2$), filled with a liquid metal characterised by a small Prandtl number ($Pr = 0.015$) and submitted to a vertical temperature gradient and an axial magnetic field. Numerical results were obtained for various values of Richardson numbers ($Ri = 0$, 0.5, 1 and 2) and Hartmann numbers $Ha = 5$, 10, 15 and 20. The objective is to determine critical Reynolds numbers for the transition from axisymmetric to non-axisymmetric flow. The critical Reynolds number Re_{cr} are observed depending on the combination of Richardson and Hartmann numbers.

2. Mathematical modelling

2.1. Model

The geometry under consideration is shown in Fig. 1. A liquid metal with a density ρ, a kinematic viscosity ν and an electrical conductivity σ, fills a cylindrical cavity of radius R and height H, submitted to a vertical temperature gradient and an axial magnetic field B. The bottom disk is rotating with a constant angular velocity Ω, and is maintained at a hot temperature T_h, while the top disk is

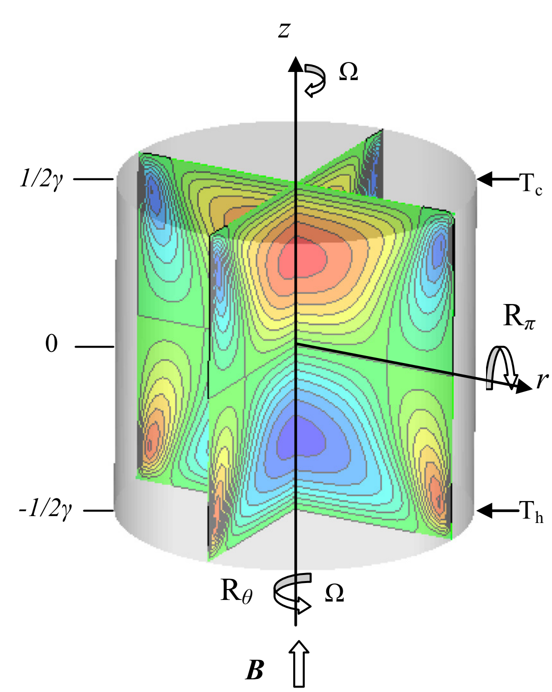

Fig. 1. Flow geometry and schematic of symmetries.