International Journal of Heat and Mass Transfer 93 (2016) 186–199

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Heat transfer enhancement using rectangular vortex promoters in confined quasi-two-dimensional magnetohydrodynamic flows

Oliver G.W. Cassells, Wisam K. Hussam, Gregory J. Sheard *

Department of Mechanical and Aerospace Engineering, Monash University, VIC 3800 Australia

A R T I C L E I N F O

Article history:
Received 7 May 2015
Received in revised form 24 September 2015
Accepted 5 October 2015

Keywords:
MHD
Quasi 2D
Hartmann
Vortex promoter
Duct flow
Channel flow
Heat transfer enhancement

A B S T R A C T

Heat transfer efficiency from a duct side-wall in which an electrically conducting fluid flows under the influence of a transverse magnetic field is investigated. A quasi-two-dimensional magnetohydrodynamic model is employed to model the flow using high-resolution numerical simulation. The gap height and angle of attack of a rectangular cylinder, with aspect ratio $\alpha = 1/2$ and blockage ratio $\beta = 1/4$, are independently varied to establish relationships between obstacle configuration and heat transfer efficiency. The heat transfer efficiency is measured through an efficiency index given by the ratio of heat transfer enhancement to pressure drop penalty in comparison to an empty duct case. At gap height ratios $1.15 \leqslant G/L_c < 2$ for an upright cylinder above a heated lower wall, thermal enhancement and efficiency can be improved; with a peak thermal efficiency of $\eta = 1.6$ occurring at $G/L_c = 1.5$. Additional increases in thermal efficiency for an obstacle at the duct centre-line ($G/L_c = 2$) can be achieved through inclining the cylinder at $\gamma = -7.5^\circ$, $\gamma = -37.5^\circ$ and $0^\circ < \gamma \leqslant 22.5^\circ$. However, these configurations offered no improvement over simply offsetting an upright cylinder at a gap height ratio of $G/L_c = 1.5$. For a cylinder offset at $G/L_c = 1.5$, varying the incidence angle through $-37.5^\circ < \gamma \leqslant 22.5^\circ$, $-7.5^\circ \leqslant \gamma < 0^\circ$ and $0^\circ < \gamma \leqslant 15^\circ$ can lead to additional thermal efficiency benefits; with a global peak efficiency of $\eta = 1.7$ occurring at $\gamma = -37.5^\circ$. The streamwise distribution of the local time-averaged Nusselt number and the effect of Hartmann dampening for $100 \leqslant Ha \leqslant 2000$ on heat transfer and flow dynamics are also investigated. A net power balance analysis reveals that in fusion applications the heat transfer enhancement dominates over the pumping power cost to produce net benefits for even modest heat transfer enhancement.

1. Introduction

Magnetohydrodynamic (MHD) flow in the presence of a transverse magnetic field occurs within the liquid metal cooling blankets surrounding magnetically confined fusion reactor plasma. Liquid metals moving under the influence of magnetic fields are subjected to electro-magnetic 'Lorentz' forces due to interactions with motion-induced electric currents [1]. Research on such flows has shown that for sufficiently strong transverse magnetic fields, velocity differentials in perpendicular planes are strongly suppressed, while vortices also become elongated and aligned with the magnetic field [1,2]. Duct walls normal to the magnetic field are subjected to the formation of boundary layers, known as Hartmann layers, which exert frictional forces on the internal core flow. For heat intensive applications, such as fusion reactor cooling blankets, this suppression of turbulent structures is detrimental to the operational efficiency, where the removal of large amounts of thermal energy is required [3]. A quasi-two-dimensional (Q2D) model can be constructed to investigate these flows by augmenting the two-dimensional Navier–Stokes equations with additional forcing and linear braking terms, representing the Lorentz forces and friction in the Hartmann layers, respectively [4].

For confined MHD flows in the absence of natural convection effects and turbulence, the validity of the quasi-two-dimensional model is only ensured when there is a sufficiently strong magnetic field (i.e. $Ha \gg 1$), the interaction parameter N, characterising the ratio of electromagnetic to inertial forces, is much greater than unity (i.e. $N = Ha^2/Re \gg 1$), and the scale of the flow structures are larger than the thickness of the out-of-plane duct wall velocity boundary layers (known as a Shercliff layers) [4,5]. Theoretical and experimental research has shown that three-dimensional instabilities in both the Shercliff layers and interior flow do exist even for large interaction parameters [4,6–11]. *Weak* homothetic forms of three dimensionality due directly to the presence of the Hartmann layers can occur in confined MHD flows for all but the highest

---

* Corresponding author.
E-mail address: Greg.Sheard@monash.edu (G.J. Sheard).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.10.006

Nomenclature

a	duct depth (out-of-plane)
B	uniform out-of-plane magnetic field strength
c_p	constant pressure specific heat capacity
$C_{d,p}$	cylinder pressure drag coefficient (x-direction)
$C_{d,v}$	cylinder viscous drag coefficient (x-direction)
C_l	cylinder lift coefficient (transverse y-direction)
Ec	Eckert number
G	gap between cylinder centroid and heated duct wall
h	duct height (in the y-direction)
h_c	cylinder cross-section short side length
H	magnetohydrodynamic friction parameter
Ha	Hartmann number
HR	heat transfer enhancement ratio, Nu/Nu_0
L_{duct}	length of duct ($L_u + L_d$)
L_c	cylinder rectangular cross-section long side length
L_d	downstream length of duct (from cylinder centroid)
L_u	upstream length of duct (from cylinder centroid)
$\mathcal{L}^2$	integral of square of velocity magnitude over computational domain
n	number of out-of-plane Hartmann layers (here $n = 2$)
N_p	element polynomial degree
Nu	time-averaged Nusselt number
Nu_0	Nu for the same duct without a cylinder
Nu_w	local Nusselt number along heated side-wall
p	pressure
P	power
Pe	Peclet number
PR	pressure drop penalty ratio, $\Delta p/\Delta p_0$
Pr	Prandtl number
Re	Reynolds number (based on $h/2$)
Re_m	magnetic Reynolds number
t	time
T	temperature scalar
T_f	bulk fluid temperature
T_o	temperature at cold inlet and top wall
T_w	temperature at hot wall
u	horizontal velocity component
u_{avg}	area-averaged horizontal velocity in duct
$\mathbf{u}$	quasi-two-dimensional velocity vector
U_0	peak fluid velocity at duct inlet
x	Cartesian coordinate (streamwise direction)
y	Cartesian coordinate (transverse direction)
z	Cartesian coordinate (vertical out-of-plane direction)

Greek symbols

α	cylinder cross-section aspect ratio
β	blockage ratio
γ	incidence angle
δ_s	Shercliff layer thickness (on duct side-walls)
ΔP	difference in power
Δp	total pressure drop from inlet to outlet
Δp_0	Δp for the same duct without a cylinder
Δt	time step size (numerical time integration)
ΔT	reference temperature difference, $T_w - T_o$
η	efficiency index, HR/PR
κ	fluid thermal diffusivity
ν	fluid kinematic viscosity
ρ	fluid density
σ	electrical conductivity of the fluid
ϕ	heat flux through heated duct side-wall
ψ	Fisher–Pearson third moment coefficient of skewness
ω	Fourier spectra frequency

magnetic field regimes. In addition, *strong* forms of MHD three-dimensionality, where only partial vortex pairing results, does occur for low Ha values [7]. Furthermore, [10] has demonstrated discrepancies between three- and two-dimensional models for interaction parameters up to $N \approx 16$. In contrast, recent research using two- and three-dimensional MHD models has shown that for high Re flows, laminarisation of the Shercliff layers still occurs for sufficiently high Hartmann numbers. Numerical work by [9] found laminarisation of the Shercliff layers at Hartmann numbers greater than 400. Furthermore, [12] demonstrated for low Re flows, three-dimensional instabilities vanished in the Shercliff layers for $Ha \geqslant 65.25$. Numerical work by [17] has also showed that modelling of vortex decay behaviour using the Q2D model is in good agreement with three-dimensional simulation data. Moreover, the quasi-two-dimensional model employed in this work has been shown to provide similar thickness scaling, instabilities, friction and turbulence behaviour to three-dimensional MHD flows [4,13–17].

Due to the dampening of velocity fluctuations along the direction of the magnetic field, longitudinal vortices are dissipated rapidly and transverse vortex generation is thus considered the most viable vortex orientation for heat transfer enhancement in ducted MHD flows [6,18,19]. Heat transfer enhancement using internal obstacles to induce transverse vortices in MHD flows has been the subject of investigation in numerous studies [19,20,22–24]. Research by [20] focused on numerically investigating the effect of Prandtl number and interaction parameter on heat transfer characteristics around a heated static circular cylinder. Furthermore, [24] focused on the effect of Reynolds number and magnetic field strength on free surface MHD flow and heat transfer behaviour around a circular cylinder. An investigation of optimal disturbances maximising energy growth in quasi-two-dimensional flow around a cylinder in a duct by [25] provided insight into the effect of Hartmann number and channel blockage on the amplification of disturbances in the vicinity of the obstacle. Subsequently, [26] promoted instability in a similar duct flow by torsionally oscillating a cylinder resulting in enhancement of heat transfer.

Few studies have investigated the variation of geometric parameters in MHD channel flow on heat transfer characteristics [19,21–23]. [19] showed that only bars placed parallel to the magnetic field aided in increasing the achievable Nusselt number in a channel flow with a heated upper wall for a variety of interaction parameters. A quasi-two-dimensional MHD numerical simulation by [22] showed that the transition from steady to unsteady flow regimes, and in turn thermal characteristics, are a function of Hartmann number and blockage ratio. Following on from the work by [22], [23] showed that there is an optimal gap height above a heated wall at which a circular cylinder should be placed to maximise heat transfer efficiency for certain blockage ratios. This optimal spacing was found to be between approximately 1 and 1.5 times the characteristic length of the obstacle. The thermal response was also shown to be a stronger function of blockage ratio than Reynolds number for a given magnetic field strength. At this location, wake vortices are cast close enough to the heated wall to enhance mixing between low and high temperature fluids, but are still cast high enough to prevent vortex suppression caused by interaction with the Shercliff layers. [21] studied the effect of Hartmann and Reynolds number on heat transfer and vortex dynamics for a square obstacle placed in a quasi-two-dimensional MHD channel. Their results showed the development of a fourth flow regime unlike that observed for hydrodynamic flows, where entrainment of the Shercliff boundary layers leads to greater flow