International Journal of Heat and Mass Transfer 93 (2016) 200–206

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Improvements of critical heat flux models for pool boiling on horizontal surfaces using interfacial instabilities of viscous potential flows

Byoung Jae Kim *, Jong Hyuk Lee, Kyung Doo Kim

Thermal-Hydraulic Safety Research Division, Korea Atomic Energy Research Institute, 111, Daedeok-daero 989, Yuseong-gu, Daejeon 34057, Republic of Korea

ARTICLE INFO

Article history:
Received 17 October 2014
Received in revised form 6 October 2015
Accepted 6 October 2015

Keywords:
Critical heat flux
Viscous potential flow
Interfacial instability
Pool boiling

ABSTRACT

Interfacial instabilities play an important role in the development of critical heat flux models. For example, the critical heat flux models for pool boiling on infinite horizontal surfaces are formulated with the aid of Rayleigh–Taylor, Kelvin–Helmholtz, and Plateau–Rayleigh instabilities for inviscid fluids. Hence, the effect of fluid viscosities is usually not included in existing models. In this study, the interfacial instabilities based on the viscous potential theory are incorporated into two representative models: a hydrodynamic model and a macrolayer dryout model. The circular jet and Kelvin–Helmholtz instabilities for viscous potential flows are utilized. The viscous potential flow approach permits a velocity slip at the interface, but it includes the effect of the viscous normal pressure. These treatments are consistent with the fact that the interface waves are induced more by pressure than by shear force. The revised models based on the viscous potential theory show better predictions. Finally, a multiplier is proposed to account for the effect of the fluid viscosities. If the value of the critical heat flux is known at atmospheric pressure, the critical heat flux at different pressures can be estimated more accurately by the multiplier.

1. Introduction

The critical heat flux represents the upper limit of efficient cooling conditions on the surface by nucleate boiling. Due to its importance in engineering applications, the critical heat flux has been extensively studied for various flows and surface conditions [1–3]. However, an exact theory of the critical heat flux has not yet been established. Steady efforts have still been made to explain the physical mechanism and improve the prediction accuracy of the models.

While reviewing the critical heat flux models for saturated pool boiling on infinite horizontal surfaces, we noticed that the effect of fluid viscosities is not included in most existing models. The absence of fluid viscosities may be attributed to the fact that inviscid flow analyses are usually performed to obtain key parameters in the models. For example, the hydrodynamic model and macrolayer dryout model utilize the Rayleigh–Taylor, Kelvin–Helmholtz, and Plateau–Rayleigh instabilities of inviscid fluids. However, if the gas and liquid viscosities are not much different, none of them can be neglected. In addition, if the thickness (or radius) of the gas layer is thin, the gas viscosity effect cannot be neglected. However, in fact, the vapor layer is thin during the film boiling so that a fully viscous flow analysis or lubrication approximation should be carried out [4,5]. Nevertheless, previous studies neglected the viscosity effect.

According to the Rayleigh–Taylor instability of viscous fluids, the vapor layer during film boiling is thin enough that the most unstable wavelength becomes $2\pi(2\sigma/(\Delta\rho g))^{1/2}$, where σ, $\Delta\rho$, and g are the surface tension, density difference between liquid and gas, and gravitational acceleration, respectively [5]. This wavelength is lower than the widely used value of $2\pi(3\sigma/(\Delta\rho g))^{1/2}$. The reduction of the most unstable wavelength was shown to slightly improve the prediction accuracy of existing critical heat flux models in a wide range of pressures [5].

Now, we note that the existing critical heat flux models make use of the Kelvin–Helmholtz instability of inviscid flows. The Kelvin–Helmholtz instability determines the maximum vapor escape velocity [6] and the initial macrolayer thickness [7]. Therefore, there is room for improving the prediction accuracy by the help of the Kelvin–Helmholtz instability of viscous fluids. The Kelvin–Helmholtz instability arises when different fluid layers are in relative motion. A uniform flow is usually considered in each fluid layer, allowing a velocity discontinuity at the interface. For this reason, a potential flow of inviscid fluids is often analyzed. However, if the viscosity effect is taken into consideration, a non-uniform flow occurs due to the shear stresses at the interface. The idea to incorporate the effect of fluid viscosities into the

---

* Corresponding author. Tel.: +82 42 868 8633; fax: +82 42 868 4801.
E-mail address: byoungjae@kaeri.re.kr (B.J. Kim).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.10.012
0017-9310/© 2015 Elsevier Ltd. All rights reserved.

Kelvin–Helmholtz instability is to use viscous potential flow theory.

The potential flow of viscous fluids satisfies the Navier–Stokes equation [8]. For the potential flow, the vorticity is identically zero so that the viscous term vanishes in the Navier–Stokes equation. As a result, the motion of a fluid can be solved using the unsteady Bernoulli equation with a velocity potential. However, the absence of the viscous term in the Navier–Stokes equation does not mean that the viscous stresses are necessarily zero. In the viscous potential theory, the shear stress is neglected at the interface and wall, allowing a velocity slip there. However, the viscous pressure enters through the normal stress balance at the interface. This treatment is consistent with the fact that the interface waves are induced more by pressure than by shear force. The viscous potential theory has been applied to various interfacial instabilities [9–11]. The viscous potential approach tends to show better predictions of the interface growth rate unless the fluid layers are too thin.

As mentioned previously, the existing critical heat flux models for saturated pool boiling on an infinite horizontal surface utilize the Kelvin–Helmholtz instability [6,7]. Because the viscous potential theory incorporates the effect of fluid viscosities into the Kelvin–Helmholtz instability, we anticipate that the theory will improve the prediction accuracy of critical heat flux models. In this study, therefore, the interfacial instabilities of viscous potential flows are applied to the development of critical heat flux models, with the aim of including the effect of fluid viscosities. Two representative critical heat flux models were revised: a hydrodynamic model and a macrolayer dryout model.

2. Critical heat flux model based on the viscous potential flow

2.1. Kelvin–Helmholtz and circular jet instabilities

Funada and Joseph [9] considered the Kelvin–Helmholtz instability for a stratified flow in which a lighter fluid overlies a heaver fluid in a gravitational field (Fig. 1a). Suppose that the gas velocity U_g is higher than the liquid velocity U_f. The critical relative velocity, $U_c = U_g - U_f$, is the maximum value for which the interface can be stable. Unless the gravity is considered, the critical relative velocity is given by

$$U_c^2 = \sigma k_c \left(\frac{\tanh(k_c\delta_g)}{\rho_g} + \frac{\tanh(k_c\delta_f)}{\rho_f} \right) \quad \text{(inviscid potential flow)}, \tag{1}$$

$$U_c^2 = \frac{\sigma k_c \left(\mu_g \coth(k_c\delta_g) + \mu_f \coth(k_c\delta_f) \right)^2}{\rho_f \mu_g^2 \coth(k_c\delta_f) \coth^2(k_c\delta_g) + \rho_g \mu_f^2 \coth(k_c\delta_g) \coth^2(k_c\delta_f)} \quad \text{(viscous potential flow)}, \tag{2}$$

where ρ, μ, δ, and k_c are the density, viscosity, fluid layer thickness, and critical wavenumber, respectively. Subscripts g and f denote the gas and liquid, respectively.

Funada et al. [10] carried out a stability analysis for a circular fluid jet into another fluid (Fig. 1b), neglecting the gravity effect. The instabilities of the circular jet are due to the capillary and Kelvin–Helmholtz instabilities. If the jet velocity relative to the surrounding fluid is zero, the situation is the same as the capillary instability. On the other hand, as the relative velocity increases, the jet undergoes the Kelvin–Helmholtz instability. Only a gas jet into liquid is considered in this study. Eqs. (4.4) and (4.16) in Funada et al. [10] are the non-dimensional critical conditions. The dimensional critical relative velocity is given by

$$U_c^2 = \frac{\sigma(\alpha_g\rho_g + \alpha_f\rho_f)}{\alpha_g\alpha_f\rho_g\rho_f} \left(k_c - \frac{1}{R^2 k_c} \right) \quad \text{(inviscid potential flow)}, \tag{3}$$

$$U_c^2 = \frac{\sigma(\beta_g\mu_g + \beta_f\mu_f)^2}{\alpha_g\rho_g(\beta_f\mu_f)^2 + \alpha_f\rho_f(\beta_g\mu_g)^2} \left(k_c - \frac{1}{R^2 k_c} \right) \quad \text{(viscous potential flow)}. \tag{4}$$

R is the gas jet diameter. α_g, α_f, β_g, and β_f are defined as

$$\alpha_g = \frac{I_0(Rk_c)}{I_1(Rk_c)}, \tag{5a}$$

$$\alpha_f = \frac{K_0(Rk_c)}{K_1(Rk_c)}, \tag{5b}$$

$$\beta_g = \alpha_g - \frac{1}{Rk_c}, \tag{5c}$$

$$\beta_f = \alpha_f + \frac{1}{Rk_c}, \tag{5d}$$

where I_0 and I_1 are the modified Bessel functions of the first kind and K_0 and K_1 are the modified Bessel functions of the second kind. For large jet diameters, α_g, α_f, β_g, and β_f approach unity. In this case, Eqs. (3) and (4), respectively, become close to Eqs. (1) and (2) with large δ_g and δ_f. In other words, the instability of a circular jet with a large diameter is equivalent to the Kelvin–Helmholtz instability with large fluid thicknesses.

2.2. Hydrodynamic theory model

Zuber [6] developed a critical heat flux model for saturated pool boiling, assuming that circular vapor jets rise at the nodes of Taylor waves (Fig. 2a). If the jet radius is a quarter of the jet spacing ($R = \lambda/4$), the critical heat flux $q_{\max}$ is given by

$$q_{\max} = \frac{\pi}{16} \rho_g L U_g, \tag{6}$$

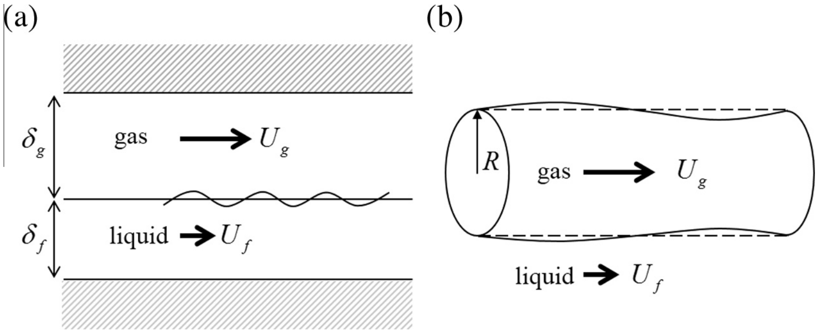

Fig. 1. (a) Two fluid layers moving parallel to each other in a two-dimensional channel. (b) A circular gas jet issuing into a liquid.