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The origin of instability in enclosed horizontally driven convection

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ABSTRACT

We demonstrate that instability in enclosed horizontally driven convection is due to a convective buoyancy-driven transverse-roll instability resembling the classical Rayleigh–Bénard convection in the thermal forcing boundary layer rather than a shear instability in the corresponding kinematic boundary layer. Instability growth is weakly sensitive to the local velocity profile, with velocity shear acting to select a transverse roll mode in preference to longitudinal rolls. The convectively unstable region grows from the hot end of the forcing boundary with increasing Rayleigh number two orders of magnitude lower than the natural onset of unstable horizontal convection. This analysis highlights the importance of the thermal boundary layer to the instability dynamics of horizontal convection, elucidating the path towards an understanding of turbulence and heat transport scaling in horizontal convection at oceanic Rayleigh numbers.

1. Introduction

The emergence of instability in enclosed horizontally driven convection (HC) — where an overturning fluid heated unevenly across a horizontal boundary transitions from a steady state — marks a key threshold in the response of this fundamental class of natural convection flows to increased strength of thermal forcing. The source of this instability remains unknown, despite its significance to debate around the existence of turbulence in horizontal convection flows, and its role in determining the scaling of horizontal convection towards oceanic scales.

Horizontal convection may contribute to global overturning in Earth's oceans, though extrapolation of the scaling between horizontal convective heat transport and thermal forcing from theory and experiment falls several orders of magnitude below accepted oceanic values. Rossby [1] argued that a balance between horizontal convection of heat within the forcing boundary layer, and vertical diffusion of heat through the forcing boundary, will produce a 1/5th-power scaling for Nusselt number (characterising convective heat transport) with Rayleigh number (characterising the strength of thermal forcing). This has been supported by experiment [2,3] and simulation [4–6], but evidence from high-resolution simulations [7] at Rayleigh numbers greater than 10^{10} has indicated that instability increases the rate of scaling, which has a theoretical upper bound of 1/3rd [4].

Here we show via a linear stability analysis applied to one-dimensional velocity and temperature profiles (obtained from high-order simulations of horizontal convection flows) that instability originates as a thermally driven instability of the boundary layer on the forcing boundary; similar analysis has proved very successful in characterising global or convective instability in extensively studied canonical flows such as Rayleigh–Bénard convection (RBC; fluid between two horizontal plates heated from below), and Rayleigh–Bénard–Poiseuille flow (RBP; RBC with a horizontal through-flow). Weber [8] showed, for a shear flow both heated from below and driven horizontally by a horizontal thermal gradient, that the preference for longitudinal or transverse rolls was dependent on the Prandtl number, stronger horizontal thermal forcing led to oscillatory instability, and that the main instability mechanism had a thermal origin for low-to-moderate horizontal thermal forcing [8–10]. Sun et al. [11] subsequently investigated the instability mechanism of HC flows. Their numerical experiment involved thermal forcing at the centre as well as side-wall forcing with two circulating cells. They concluded that velocity shear instability rather than thermal instability is responsible for the unsteady HC flow through a Hopf bifurcation with a critical Rayleigh number of 5.5377×10^8 at Prandtl number $Pr = 1$. In contrast to [11], we consider HC in water [12–14] with a single overturning cell, and a Prandtl number $Pr = 6.14$. At this Prandtl number, King [7] showed that horizontal convection in enclosures with height-to-length aspect ratio $H/L \geqslant 0.16$ driven by a linearly increasing temperature profile along the base became unstable to unsteady flow at $3.5 \times 10^8 \lesssim Ra \lesssim 8.5 \times 10^8$.

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Nomenclature

$\mathbf{A}$	generalised eigenvalue matrix (left-hand side)
$\mathbf{B}$	generalised eigenvalue matrix (right-hand side)
D	operator representing partial derivative with respect to y
f	generic symbol representing a horizontally parallel flow variable (e.g. velocity, pressure or temperature)
$\tilde{f}$	generic symbol representing a perturbation flow variable
f_B	generic symbol representing a horizontally parallel base flow variable
g	gravitational acceleration
$\hat{\mathbf{g}}$	unit vector in direction of gravity
H	enclosure height
i	imaginary unit
$\tilde{\theta}$	eigenfunction of infinitesimal temperature perturbation
k	total wavenumber, $k^2 = \alpha^2 + \beta^2$
L	enclosure width; characteristic length of thermal forcing for horizontal convection
p	pressure
$\tilde{p}$	eigenfunction of infinitesimal pressure perturbation
p_B	pressure, base flow
Pr	Prandtl number, $Pr = \nu/\kappa$, here $Pr = 6.14$ throughout
Ra	Rayleigh number based on imposed temperature difference across heated horizontal boundary
Ra_c	critical Rayleigh number
$Ra_{c,\mathrm{m}}$	critical marginal Rayleigh number
t	time
$\mathbf{u}$	velocity vector
u	horizontal velocity component
$\tilde{u}$	eigenfunction of infinitesimal horizontal velocity perturbation
u_B	horizontal velocity component, base flow
v	vertical velocity component
$\tilde{v}$	eigenfunction of infinitesimal vertical velocity perturbation
w	transverse (out-of-plane) velocity component
$\tilde{w}$	eigenfunction of infinitesimal transverse velocity perturbation
x	Cartesian horizontal coordinate
$\mathbf{x}_k$	eigenvector, concatenation of collocation-point values of $\tilde{v}$ and $\tilde{u\theta}$
y	Cartesian vertical coordinate
z	Cartesian transverse (out-of-plane) coordinate

Greek symbols

α	travelling wave number in horizontal (x) direction
α_c	critical horizontal travelling wave number
α_T	volumetric thermal expansion coefficient
β	travelling wave number in transverse (z) direction
δ	an arbitrary small constant
δ_θ	temperature difference imposed across horizontal boundary
κ_T	fluid thermal diffusivity
λ_{1D}	predicted horizontal wavelength of instability from 1D linear stability analysis
λ_{2D}	horizontal wavelength of disturbance from two-dimensional simulation
ν	fluid kinematic viscosity
θ	fluid temperature
θ_B	fluid temperature, base flow
ω	complex eigenvalue representing growth rate and frequency of an instability eigenmode
ω_i	imaginary part of complex eigenvalue ω
ω_r	real part of complex eigenvalue ω

More recently, Gayen et al. [6] used a mechanical energy budget to explain the transition of horizontal convection from small scales of motion driven mainly by thermal convection to a shear instability of the large-scale flow at high Rayleigh number. The three-dimensional direct numerical simulation in that study was carried out in an enclosure with aspect ratio $H/L = 0.16$ at a Prandtl number $Pr = 5$, with horizontal convection driven by a step-change in temperature at half the horizontal distance along the base. The same setup was considered at a range of Rayleigh numbers in [15], elucidating a complex instability pathway for horizontal convection: stable, laminar overturning flow was produced at Rayleigh number $Ra = 5.86 \times 10^7$, while at $Ra = 5.86 \times 10^8$ unsteady two-dimensional (transverse) rolls were detected in the boundary layer near the hot end of the enclosure. At a higher Rayleigh number $Ra = 5.86 \times 10^9$, these structures were more closely spaced and visible from further upstream, and at $Ra = 5.86 \times 10^{10}$ and 5.86×10^{11} the two-dimensional structures were superposed by longitudinal-roll structures appearing at approximately mid-way along the base. The longitudinal structures were dominant at the higher Rayleigh number, and began merging and interacting approximately two-thirds of the distance along the base, before erupting into mushroom plumes closer to the end-wall. These findings point to the source of instability in horizontal convection as a convective instability in the boundary layer, and the present work aims to elucidate this instability mechanism via a local one-dimensional (1D) linear stability analysis. We will show that this analysis reveals the instability to be thermally driven.

2. Numerical setup

The system comprises a rectangular enclosure of width L and height H aligned with Cartesian coordinates x and y, respectively, with z the transverse coordinate. The flow is driven by a time-invariant temperature profile increasing linearly in x imposed along the bottom of the enclosure. The side and top walls are insulated (zero normal gradient of temperature), and a no-slip (zero velocity) condition is imposed on the velocity field on all walls. Taking the temperature difference imposed across the forcing boundary $\delta\theta$, volumetric expansion coefficient α_T, gravitational acceleration g, kinematic viscosity ν and thermal diffusivity κ_T, the characteristics and strength of this circulation are determined by a Rayleigh number $Ra = \alpha_T g\,\delta\theta L^3/\nu\kappa_T$ and Prandtl number $Pr = \nu/\kappa_T$ characterising the strength of thermal forcing and the ratio of molecular to thermal diffusivity, respectively. We adopt a Boussinesq approximation for buoyancy, in which density differences in the fluid are neglected except through the gravity term in the momentum equation. Under this approximation the energy equation reduces to a scalar advection–diffusion equation for temperature which is evolved in conjunction with the velocity field governed by the incompressible Navier–Stokes equations. Introducing velocity vector $\mathbf{u}$ with components u, v and w respectively in x, y and z, a pressure p and temperature θ, and normalising length, velocity, time, pressure and temperature by $L, \kappa_T/L, L^2/\kappa_T, \rho\kappa_T^2/L^2$ and $\delta\theta$ permits the governing equations to be expressed as

$$\partial_t\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + Pr\,\nabla^2\mathbf{u} - PrRa\,\hat{\mathbf{g}}\,\theta, \tag{1}$$