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Subcritical and supercritical bifurcations for two-phase flow in a uniformly heated channel with different inclinations

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ABSTRACT

In the past few decades, the stability of two-phase flow in vertical channels has been studied quite extensively. However, such studies for horizontal channels are relatively limited. Moreover, stability analysis of inclined channels is not available in the literature on two-phase flows. Therefore, linear stability analysis using a reduced order model for two-phase flow in inclined channels is carried out. In addition, bifurcation analysis is also carried out to capture the non-linear dynamics of the system and to identify the regions in parameter space for which subcritical and supercritical bifurcations exist. The study is carried out for different inclination angles in order to characterize the effect of inclination on the stability of the system. It is observed that the stability characteristics of two-phase flow in horizontal and inclined channels are significantly different from those of vertical channels even for the same operating conditions. Furthermore, from the non-linear analysis it is seen that the dynamics of the system for inclined and horizontal channels are quite complex. At an inclination of 45°, three generalized Hopf (GH) points (these points separate subcritical region from supercritical region) have been observed in the stability map. These GH points do not exist for vertical channels. Moreover, numerical simulations of the time-dependent, non-linear ODEs are carried out for the selected points in the operating parameter space to verify the stability behavior as predicted by stability maps.

1. Introduction

The stability of two-phase flow in a uniformly heated channel is important in several industrial domains like refrigeration systems, turbo-machinery, boiling water reactors, solar steam generating systems and two-phase flow heat exchangers [1]. The orientation of the channels for these applications can be horizontal or vertical. For example, the fuel bundle arrangement in Canada's CANDU reactors are horizontal, while the Euratom (Europe and Japan) contribution to the Generation IV International Forum is the high-performance light water reactor, which has a vertical orientation of its core [2]. From the literature review it is seen that the instability in vertical channels is extensively studied [1,3–11]. However, only a few researchers have investigated instabilities in horizontal channels [12–16,23]. Different methods have been proposed to analyze the instability in such systems [2,12]. The investigation reported different types of instabilities which may occur in such non-linear systems. It is noted that detailed studies on the effect of inclination on the stability of the channel are not reported in literature.

Thermally induced oscillations of the flow rate and system pressure are undesirable, as they can cause mechanical vibrations, thermal fatigue, problems of system control and in extreme circumstances disturb the heat transfer which causes the occurrence of thermal crisis. The evaluation of the instability threshold values permits to determine the safe operating regions of a two-phase flow system [17]. Mainly three types of instability are studied in such systems namely, Ledinegg, pressure drop oscillations and Density Wave Oscillations (DWO). Ledinegg instability involves a sudden change in the flow rate to a lower value. It occurs when the slope of the demand pressure-drop versus flow rate curve (internal characteristic) of the channel is negative and steeper than that of the loop supply pressure drop versus flow rate curve (external characteristic) and there are multiple intersections of the internal and external characteristics [12]. Pressure drop instability occurs due to flow excursion which initiates dynamic interactions between the channel and the compressible volume [12–15].

The most extensively studied type of instability in two-phase flow systems is DWO, which occurs due to the perturbation in

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Nomenclature

A	cross sectional flow area (m^2)
$a_i\,(t)$	phase variable related to single phase enthalpy
D	diameter of the channel (m)
f	friction factor (0.188 $Re^{-0.22}$)
Fr	Froude number
h_{in}	coolant inlet enthalpy (J/kg^{-1})
h_{sat}	coolant saturation enthalpy (J/kg^{-1})
Δh_{fg}	vapor–liquid enthalpy difference (h_g–h_f)
k_{exit}	exit pressure loss coefficient
k_{in}	inlet pressure loss coefficient
L	flow channel length (m)
N_f	friction number
N_{pch}	phase change number
N_{sb}	subcooling number
q''	wall heat flux (W m^{-2})
$s(t)$	phase variable related to two-phase quality
v	coolant velocity (m s^{-1})
x	quality
z	channel axial spatial coordinate (m)

Greek letters

ξ	perimeter of channel (m)
λ	boiling boundary (m)
ΔP_{ext}	external pressure drop (N m^{-2})
Θ	inclination angle form the horizontal
ρ	density (kg/m^3)

Subscripts

1ϕ	single phase
2ϕ	two-phase
acc	acceleration
$exit$	channel exit
ext	external
f	liquid (water)
g	vapor
in	channel inlet
m	mixture

Superscripts

–	fixed point value
*	dimensional quantity

the inlet flow rate of a coolant which changes the local vapor generation rate and hence the two-phase mixture density as well. This perturbation propagates from the inlet to the outlet of the channel, creating a disturbance in the local pressure drop which may in-turn intensify the flow rate perturbation. This self-amplified thermal hydraulic feedback produces the so-called DWO [3,7,19]. It is noted that for many engineering systems (such as solar thermal and nuclear) operating pressures are often quite high. Since at high pressures only the DWO instability is observed, the present analysis focuses on this type of instability.

The system studied in the present work is modeled as a set of coupled non-linear equations. The stability of coupled non-linear equations have been studied extensively by many researchers [11,18–22]. It is well known that non-linear systems show complex behavior under specific conditions. For example, decaying and non-decaying oscillations were observed in systems like boiling water reactors, theoretically as well as experimentally [20]. From the operational point of view, it is important to identify the regions in the parametric space, in which the system can become unstable. The linear stability analysis, which is valid for very (infinitesimally) small perturbations is not sufficient for this purpose as perturbations can be relatively large. The bifurcation analysis can locate the regions in the parameter space which have unstable limit cycles, even in the regions identified as stable by linear stability analysis. These are the regions where the type of the Hopf bifurcation is subcritical. Similarly, the supercritical Hopf bifurcation leads to a stable limit cycle in the (linearly) unstable regions. These regions can be identified by non-linear stability analysis. The identification of the subcritical region is very important for understanding the stability of the system for obvious reasons.

The objective of the present work is to carry out linear stability analysis and bifurcation analysis of uniformly heated channels at various inclinations. Using non-linear analysis, generalized Hopf points are identified, which demarcate the boundary between subcritical and supercritical Hopf bifurcations. Moreover, numerical simulations are carried out to verify and understand the characteristics of the system.

2. Description of the model

2.1. Description of the flow channel

Fig. 1 represents the flow channel having a length which is heated uniformly. The single phase coolant enters from the bottom of the channel with a velocity of v_{in} and starts boiling at a certain point inside the channel (i.e. after the inlet). This point is the boiling boundary, beyond which the coolant continues as a two-phase mixture of liquid and its vapor. The coolant enters the flow channel at an inlet temperature, and the temperature increases in the single phase region reaching the saturation temperature at the boiling boundary. Beyond the boiling boundary the temperature of the coolant remains the same (although quality increases). The angle θ in the figure defines the orientation of the channel, it varies from 0° to 90° i.e. from a horizontal to a completely vertical channel. During the whole analysis, the operating pressure of the system

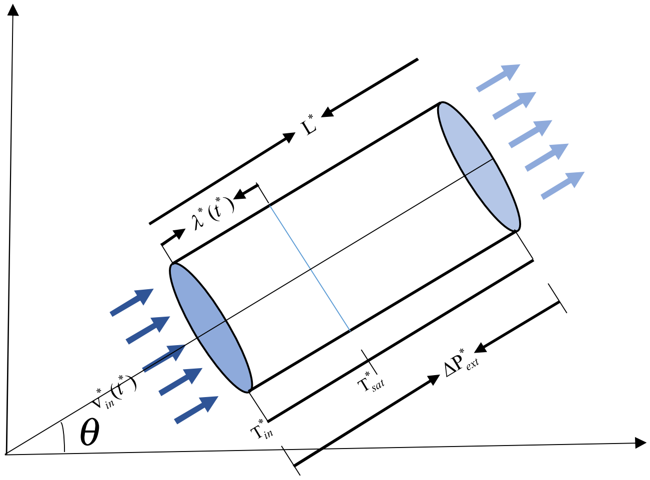

Fig. 1. Schematic of the Inclined heated channel.