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Review

The singular boundary method for steady-state nonlinear heat conduction problem with temperature-dependent thermal conductivity

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ABSTRACT

This paper presents the singular boundary method for steady-state nonlinear heat conduction problems. In the steady-state nonlinear heat conduction problem, the Kirchhoff transformation is employed to remove the nonlinearity associated with the temperature dependence of the thermal conductivity. Then the transformed Laplace-type equation is investigated by the present singular boundary method with a simple iteration procedure. Finally, the temperature field is derived by the inverse Kirchhoff transformation. The present algorithm is verified on several examples involving various expressions of temperature dependent thermal conductivity and different computational domains. Numerical results show good accuracy and stability of the proposed strategy.

Contents

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Nomenclature

x_1, x_2 co-ordinates [m]
T temperature [K]
q heat flux [W/m^2]
k thermal conductivity [W/m^2]
h heat transfer coefficient [W/(m^2 K)]
R contact resistance [m^{2}K/W]
tol tolerance
$iter$ number of iteration
s_j source points
x_m collocation point
x_I inner point
n outward normal to the boundary
n_x outward unit vector to the boundary at the collocation point
n_s outward unit vector to the boundary at the source point
N number of source points
M number of test nodes
Q^{jj}, U^{jj} source intensity factors
$G(s_j,x_m)$ fundamental solution
L length [m]
δ_{RMSE} relative root mean square error
$Cond$ condition number
m power parameter
CPU calculation time [s]
s dimensionless length, measured along the circumference

Greek symbols
Ω computational domain
Γ boundary
α, β, γ, ε, ξ constants
θ temperature in the Kirchhoff space
ψ function of Kirchhoff transformation
μ interchange factor
σ Stefan–Boltzmann constant [W/(m^2 K^4)]

Subscripts
l left
r right
f fluid
b boundary
D boundary with Dirichlet condition
N boundary with Neumann condition
R boundary with Robin condition
1 first subregions
2 second subregions
num numerical
$exact$ exact

1. Introduction

In many industrial processes, it is necessary and justification to assume that the material-physical properties are not constant and depend on the temperature. In most of the semiconductor materials, the effect of temperature-dependent thermal conductivity contributes an additional temperature rise and should be accounted for in thermal analysis of GaN-based electronics [1]. The assumption that the thermal conductivity depends on temperature may contribute to a better understanding in many industrial process, for instance the hot-stamping process [2] and the sophisticated thermal management. The ignorance of temperature-dependent of the thermo-physical parameters assumptions violates the actual fins operating conditions. Fins are used to increase the heat transfer of heating systems such as, cooling electric transformers, cooling of computer processor, IC engines, air conditioning and refrigeration. In the heat transfer problems in high-temperature environments or if large temperature differences exists within a fin the assumption of the temperature dependence of the thermal conductivity is necessary [3–10]. This assumption leads to nonlinearity of governing equation. In the steady-state case, this nonlinearity can be removed by employing the Kirchhoff transformation. The original nonlinear partial differential equation (PDE) is replaced by the Laplace equation in the transformed space. The boundary conditions of first and second kind pose no problem for the transformation, but the third kind boundary condition and the interface condition become nonlinear. However this nonlinearity is not strong which may cause the convergence problems.

For such nonlinear heat transfer problem the approximate results can be easily obtained by numerical methods. One-dimensional nonlinear heat conduction problem has been solved with some semi analytical methods, such as the perturbation method (PM) [11,12], the variational iteration method (VIM) [13], the homotopy analysis method (HAM) [14], the differential transform method (DTM) [3–6,15,16] and the Adomian decomposition method (ADM) [8,17]. Two-dimensional boundary value problems have been the subject of several studies using the ADM [9], the finite element method (FEM) [18–20], the boundary element method (BEM) [21–23], the method of fundamental solutions (MFS) [24,25], the fundamental solution-based hybrid finite element method (HFS-FEM) [26], the hybrid Trefftz finite element method (HT-FEM) [27] or the boundary knot method (BKM) [28,29]. In [10], homotopy perturbation sumudu transform method (HPSTM) has been used to evaluate temperature distribution and effectiveness of radial fins with temperature-dependent thermal conductivity and exposed to convection.

Like the BEM, the MFS is applicable when a fundamental solution of the operator governing the PDE is explicitly known. The MFS is much easier to implement than the BEM since no surface integrals need to be calculated. In the MFS, the solution of the problem is approximated by a linear combination of fundamental solutions with the singularities located outside the solution domain on a fictitious boundary. However, the MFS has the disadvantage of the distribution of the source points. The singularities may be either pre-assigned, or let free and determined as part of the solution of the problem. In [25] in order to determine the optimal positioning of fictitious-boundaries, authors use the minimization of the maximum error in the boundary conditions.

As an alternative to the MFS, Chen and Wang proposed a novel numerical method, called the singular boundary method (SBM) [30]. The SBM is mathematically simple, easy-to-program, accurate, meshless and integration-free and avoids the controversy of the fictitious boundary in the MFS. Recently, the SBM has been successfully implemented in heat conduction problems [31,32] including anisotropic materials [33,34], acoustic problem [35,36], elastostatic problems [37] and water wave problems [38,39] and so on. In [33] the SBM was proposed in solving steady state heat conduction problem in three-dimensional anisotropic materials with the symmetric and positive-definite thermal conductivity coefficients. In [34], authors investigated the efficiency of the SBM for the solution of Cauchy steady-state heat conduction problems in an anisotropic medium. In both papers [33,34] the SBM is