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Precise time-domain expanding dual reciprocity boundary element method for solving transient heat conduction problems

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ABSTRACT

The dual reciprocity boundary element method (DRBEM) and the precise time-domain expanding algorithm are combined for solving transient heat conduction problems with heat sources. Firstly, the recursive formulation of the governing equation is derived by expanding time-dependent physical quantities at any discrete time interval. Then, the recursive equation is solved by the DRBEM, while a self-adaptive check scheme is used for estimating recursive times in a time step. Finally, the single and multi-connected domain problems are analyzed respectively with different kinds of boundary conditions. The results show that the proposed method can obtain very stable and accurate results with different time steps.

1. Introduction

It is well known that many practical engineering problems are closely related to the transient heat transfer process. Exploring distribution of physical quantities such as temperature and heat flux is very important, which is closely concerned with the safety and cost of engineering. Recently, numerical schemes have become the most important and popular methods due to the complexity of transient heat conduction problems. Generally, the numerical methods [1–3] mainly include the finite difference method (FDM), the finite element method (FEM), the finite volume method (FVM), the meshless method and the boundary element method (BEM). For most methods, the finite difference is often adopted to replace the derivative of time for time-dependent problems. Although the finite difference is simple and convenient, numerical results fluctuate according to the change of time steps. It is an urgent task to seek a time-domain processing method to ensure the stability and convergence of results.

In 1999, Yang [4] presented the precise time-domain expanding algorithm, which can not only obtain the stable and accurate numerical results, but also check the needed number of expansion terms by a self-adaptive technique. Up to now, the method combining the precise time-domain algorithm with FEM or Meshless method has been applied to many fields, such as heat transfer problems [4] and viscoelastic problems [5–7].

Compared with FDM, FEM, FVM and Meshless method, BEM is a very robust analysis method for solving the linear and homogeneous heat conduction problems [8–11] based on the known fundamental solution of problems. However, it is very difficult for complex transient heat conduction problems to obtain the fundamental solution except some very special cases [12–15]. Fortunately, we can adopt the fundamental solution of the approximate problem to establish the boundary integral equation, whereas domain integrals will be involved in the resulting integral equation. In order to keep the advantage of reducing dimensionality of BEM, the domain integrals are necessary to transform into the boundary integrals.

Many excellent transformation methods of domain integrals have been presented by BEM researchers, such as the dual reciprocity method (DRM) [16] and the radial integral method (RIM) [17,18]. In general, DRM requires particular solutions which restricts its application to complicated problems. However, when the particular solutions of problems can be obtained through simple approximation functions, some complicated problems can be solved perfectly. RIM is a pure mathematical treatment without using the particular solutions of problems, whereas many radial integrals need to be computed. Particularly, the computation of radial integral is time-consuming for the large number of nodes. The dual reciprocity BEM (DRBEM) and the radial integral BEM (RIBEM) are obtained by combining DRM and RIM with BEM, respectively. The two methods have been widely applied to many fields including the natural convective flow of micropolar fluid problem [19], the inverse natural magneto-convection problem [20], the magneto-thermo-viscoelastic problem [21], the dynamic

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Nomenclature

A	coefficient matrix
c	specific heat
f	heat source
$f^{(m)}$	expansion coefficient of f with order m
G	weight function
k	thermal conductivity
N_b	number of boundary nodes
N_I	number of internal nodes
N_t	total number of nodes
q	heat flux
$q^{(m)}$	expansion coefficient of q with order m
q	heat flux vector
t	time
T	temperature
T_0	initial temperature
$T^{(m)}$	expansion coefficient of T with order m
T	temperature vector
x, **y**	nodes
x_1, x_2	Cartesian coordinates of the node x
X	unknown vector
Y	known right-hand-side vector
∇^2	Laplace operator
ρ	density
Ω	domain of problem
Γ	boundary of the domain Ω
Δt	time step
$\phi(R)$	radial basis function
$\Vert \bullet \Vert_2$	vector 2-norm
ε	error bound

Subscripts

b	boundary node
I	internal node

Superscripts

m	order of expansion

Abbreviations

DRBEM	dual reciprocity BEM
PTE-DRBEM	precise time-domain expanding and DRBEM
FD-DRBEM	finite difference and DRBEM

analysis of laminate composite plate [22], the nonlinear and non-homogeneous elastic problem [23], the crack analysis in functionally graded materials [24], the viscous flow problem [25] and the heat conduction problem [26]. But solutions are sensitive to different time steps due to using the finite difference technique to replace the derivative term with respect to time. In 2014, Yu et al. [27,28] combined the precise time-domain expanding method with RIBEM to solve the transient heat conduction problems.

There is a history of over 30 years for DRBEM since it was proposed in 1983 [16]. Even now, the method still shows strong vitality [29]. The time difference method is widely used to deal with the time-domain when using DRBEM. However, numerical results are sensitive to different time steps, which still exists in DRBEM.

In this paper, the precise time-domain expanding and the DRBEM (PTE-DRBEM) are combined to solve transient heat conduction problems. By expanding the time-dependent quantities in discrete time intervals, the DRBEM recursive formulation is derived with self-adaptive check technique to improve the computational accuracy. Finally, some numerical examples are shown to validate the present method.

2. Governing equation

For the isotropic media, the governing equation of transient heat conduction problems with constant physical parameters can be given by

$$k\nabla^2 T(\mathbf{x},t) + f(\mathbf{x},t) = \rho c \frac{\partial T(\mathbf{x},t)}{\partial t}, \quad \mathbf{x} \in \Omega \tag{1}$$

where $\mathbf{x} = (x_1, x_2)$, Laplace operator $\nabla^2 = \partial^2/\partial(x_1)^2 + \partial^2/\partial(x_2)^2$, $T(\mathbf{x},t)$ is the temperature of point $\mathbf{x} \in \Omega$ at time t, $f(\mathbf{x},t)$ is the heat source, k, ρ and c are thermal conductivity, density and specific heat, respectively.

The initial condition can be expressed as

$$T(\mathbf{x},0) = T_0(\mathbf{x}) \tag{2}$$

where T_0 is a prescribed function. In present study, two type boundary conditions will be considered including the temperature and the heat flux. The boundary conditions can be written as

$$\begin{cases} T(\mathbf{x},t) = \bar{T}(\mathbf{x},t) & \mathbf{x} \in \Gamma_1 \\ q(\mathbf{x},t) = \bar{q}(\mathbf{x},t) & \mathbf{x} \in \Gamma_2 \end{cases} \tag{3}$$

where $\Gamma_1 \cup \Gamma_2 = \Gamma, \Gamma_1 \cap \Gamma_2 = \varnothing$, $\Gamma = \partial\Omega$, q is the heat flux and can be expressed as $q = -k\partial T/\partial \mathbf{n}$, $\bar{T}$ and $\bar{q}$ are prescribed temperature and heat flux history on the corresponding boundary, respectively.

3. Recursive governing equation in a discrete time-domain

In a discrete time interval $[t_l, t_{l+1}]$, $T(\mathbf{x},t)$, $q(\mathbf{x},t)$ and $f(\mathbf{x},t)$ at time t_l can be expanded in the following Taylor series forms [4,27,28]

$$T(\mathbf{x},t) = \sum_{m=0} T^{(m)}(\mathbf{x}) s^m(t) \tag{4}$$

$$q(\mathbf{x},t) = \sum_{m=0} q^{(m)}(\mathbf{x}) s^m(t) \tag{5}$$

$$f(\mathbf{x},t) = \sum_{m=0} f^{(m)}(\mathbf{x}) s^m(t) \tag{6}$$

where $s = (t - t_l)/\Delta t_{l+1} (l = 0, 1, 2, \ldots)$ and $\Delta t_{l+1} = t_{l+1} - t_l$ is the $(l+1)$th time step. $T^{(m)}$, $q^{(m)}$ and $f^{(m)}$ are the mth expansion coefficients of T, q and f respectively. They are only related to the spatial coordinates **x**. The expansion coefficients from the standard Taylor series can be obtained by

$$(\cdot)^{(m)} = \frac{(\Delta t_{l+1})^m}{m!} \frac{\partial^m(\cdot)}{\partial t^m}\bigg|_{t=t_l} \tag{7}$$

where $(\cdot)$ denotes the functions T, q or f. The derivative of $T(\mathbf{x},t)$ with respect to t and **x** can be respectively expressed as

$$\frac{\partial T(\mathbf{x},t)}{\partial t} = \sum_{m=0} \frac{m+1}{\Delta t_{l+1}} T^{(m+1)}(\mathbf{x}) s^m(t) \tag{8}$$