International Journal of Heat and Mass Transfer 93 (2016) 665–673

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A general method for solving transient multidimensional inverse heat transfer problems

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ARTICLE INFO

Article history:
Received 4 August 2015
Received in revised form 11 September 2015
Accepted 11 September 2015

Keywords:
Inverse problems
Inverse method
Heat conduction
Heating optimization

ABSTRACT

The purpose of this work is to formulate a simple method, which can be used for the solution of an inverse transient heat flow problem. The proposed algorithm allows to reconstruct the transient temperature distribution in a whole structure element based on measured temperatures at selected points on the outer surface or inside in the component. The presented method allows for the use of commercial calculation programs. In this work, ANSYS Multiphysics software, that utilizes finite element method (FEM) is applied. Both linear and nonlinear finite elements have been used. The unknown boundary condition can be assumed as a staircase function or a polynomial function. Due to the implementation of the finite element method, the proposed method can be applied to complex-shape-bodies with temperature-dependent thermophysical properties. Numerical examples of the identification of transient heat conduction and simultaneous transient heat flow by conduction and radiation will be presented. The presented method allows to optimize the power block's shut-down and start-up operations. It also contributes to the reduction of heat loss during these operations and the extension of the power block's life span.

1. Introduction

Inverse problems, in contrast to direct boundary value problems, are characterized by an unknown part of boundary conditions. It happens very often in engineering, because of the difficulties in the definition of some boundary conditions. The identification of the unknown boundary heat flux for an atmospheric reentry capsule is presented in [1]. Searching the temperature and heat flux on the inner edge of a gas turbine blade is shown in [2]. The reconstruction of a convective boundary condition on the inner surface of a steam header is presented in [3]. The formulation of a transient inverse heat conduction problem and a general method to solve it can be found in [4,5]. The ill-posed boundary problem is solved by introducing additional temperature measurements to the analysis. Boundary conditions are so chosen that the calculated temperatures are close to the measured temperatures. Application of the optimization theory allows to formulate methods working in the off-line mode. Space marching methods [3] are so fast that they can solve the problem in the on-line mode. Ill-posed problems are sensitive to random measuring errors and therefore, in order to overcome such difficulties a variety of techniques have been proposed in the following literature: regularization [4], future time steps [6] and smoothing digital filters [3].

The solution of inverse problems often requires the use of a sophisticated mathematical model [1–5]. For this reason, the identification of transient processes is hard to perform for many engineers. Commonly used commercial programs for heat flow modeling are difficult to apply, because they require the definition of all boundary conditions.

The aim of this work is the proposition of a simple method, which can be used for the solution of inverse transient heat flow problems. The proposed algorithm allows for the use of commercial calculation programs. In this work, ANSYS Multiphysics software [7] that utilizes finite element method (FEM) is applied. Numerical examples of the identification of transient heat conduction and simultaneous transient heat flow by conduction and radiation will be presented.

2. Formulation of the problem

The equation governing the transient heat conduction problem in a solid is given by

$$c(T)\rho(T)\frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q} + q_V, \tag{1}$$

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.09.029
0017-9310/© 2015 Published by Elsevier Ltd.

where q_V is the heat generation rate per unit volume, $\mathbf{q}$ is the heat flux vector defined by Fourier's law

$$\mathbf{q} = -\mathbf{D}\,\nabla T. \tag{2}$$

$\mathbf{D}$ is the conductivity matrix and it can be written as

$$\mathbf{D} = \begin{bmatrix} k_x(T) & 0 & 0 \\ 0 & k_y(T) & 0 \\ 0 & 0 & k_z(T) \end{bmatrix} \tag{3}$$

If material is isotropic then $k_x(T) = k_y(T) = k_z(T) = k(T)$. All material properties (c – specific heat, k – thermal conductivity, ρ – density) are considered as known functions of temperature.

Transient heat conduction problems are initial-boundary problems for which one is required to assign appropriate initial and boundary conditions. Initial condition, also called Cauchy condition, is the temperature value of a body at its first moment t_0 = 0 s.

$$T(\mathbf{r}, t)|_{t_0=0} = T_0(\mathbf{r}) \tag{4}$$

where $\mathbf{r}$ is position vector.

Three of the most often used boundary conditions, 1st, 2nd and 3rd kinds, can be assigned to the body's boundary

$$T|_{\Gamma_T} = T_b \tag{5}$$

$$(\mathbf{D}\,\nabla T \cdot \mathbf{n})|_{\Gamma_q} = q_B \tag{6}$$

$$(\mathbf{D}\,\nabla T \cdot \mathbf{n})|_{\Gamma_h} = h(T_m - T|_{\Gamma_h}) \tag{7}$$

where

$\mathbf{n}$ – unit outward normal vector to the boundary Γ,
T_b – temperature set on the body boundary Γ_T,
q_B – heat flux set on the body boundary Γ_q,
h – heat transfer coefficient set on the body boundary Γ_h,
T_m – temperature of a medium.

A phenomenon described by Eqs. (1)–(7) in which boundary conditions are specified over the entire boundary is called a direct transient heat conduction problem (Fig. 1).

If boundary conditions (5)–(7) are unknown on parts of the domain boundary, the problem becomes ill-posed and additional interior temperature measurements are needed in the analysis (Fig. 2)

$$f_i(t) = T(\mathbf{r}_i) \quad i = 1, \ldots, N_T \tag{8}$$

A boundary with unknown conditions, is discretized into N_u intervals and is approximated by the 2nd kind boundary condition (6). It can be assumed as a staircase or a polynomial function on the body surface (Fig. 3).

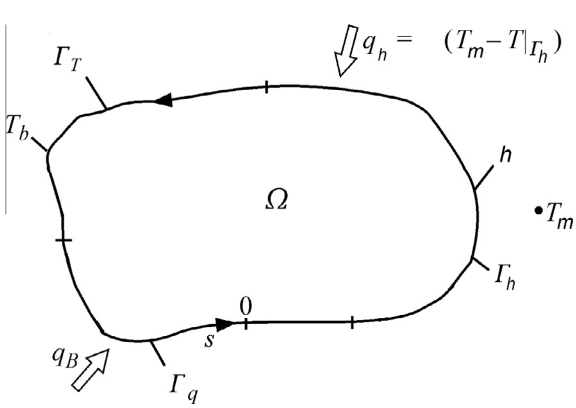

Fig. 1. A direct transient heat conduction problem.

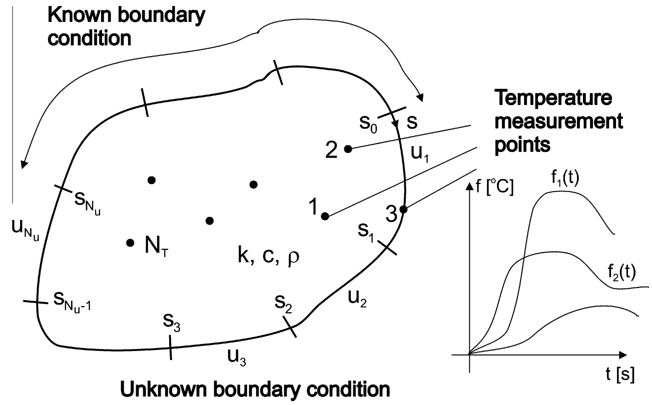

Fig. 2. An inverse problem with known, unknown boundary conditions and additional temperature measurement points.

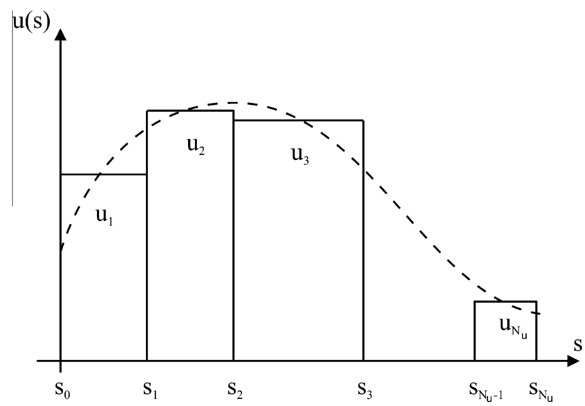

Fig. 3. Staircase or polynomial approximation of unknown heat flux on the body surface.

The aim is to choose such $\mathbf{u}(u_1(t), u_2(t), \ldots, u_{N_u}(t))$ in time that the computed temperatures would agree within the certain limits with the experimentally measured temperatures. It may be expressed as

$$T_i(\mathbf{u}, \mathbf{r}_i, t) - f_i(t) \cong 0, \quad i = 1, \ldots, N_T \tag{9}$$

The number of measurement points N_T should be larger than the number of unknown boundary conditions N_u.

It's hard to solve the Ill-posed problems. They are very sensitive to random measuring errors in input data. Many methods can be used to stabilize the solution. In this paper, the future time steps and smoothing filters are used. To stabilize the solution, vector $\mathbf{u}_L$ is held constant in the N_F future time steps (Fig. 4)

$$\mathbf{u}_L = \mathbf{u}_{L+1} = \ldots = \mathbf{u}_{L+N_F-1} \tag{10}$$

where $\mathbf{u}_L = \mathbf{u}(u_1(t_L), u_2(t_L), \ldots, u_{N_u}(t_L))$.

The least-squares method is used to determine parameters $\mathbf{u}_L$. The sum of squares

$$S = \sum_{i=1}^{N_T} \sum_{j=L}^{L+N_F-1} \left[f(t_j)_i - T(\mathbf{u}_L, \mathbf{r_i}, t_j)\right]^2 \tag{11}$$

can be minimized by a general unconstrained method.

However, the properties of (11) make it worthwhile to use methods designed specifically for the nonlinear least-squares problem. In this paper, the Levenberg–Marquardt method [8] is used to determine the parameters $\mathbf{u}_L$.