International Journal of Heat and Mass Transfer 91 (2015) 235–245

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Thermal shock fracture of a cylinder with a penny-shaped crack based on hyperbolic heat conduction

S.L. Guo [a,*], B.L. Wang [a,b]

[a] Graduate School at Shenzhen, Harbin Institute of Technology, Harbin 150001, PR China
[b] Institute for Infrastructure Engineering, University of Western Sydney, Penrith, NSW 2751, Australia

ARTICLE INFO

Article history:
Received 17 April 2015
Received in revised form 22 July 2015
Accepted 22 July 2015

Keywords:
Fracture mechanics
Cylinder specimen
Thermal shock
Hyperbolic heat conduction

ABSTRACT

This paper studies the thermal shock fracture of a cracked cylinder based on the hyperbolic heat conduction. The crack faces are subjected to a sudden anti-symmetric thermal flux and a sudden symmetric thermal flux, respectively. By Laplace transform and dual integral equation technique, the mode II stress intensity factor and the mode I stress intensity factor are developed at the crack front for the two cases, respectively. Numerical results of stress intensity factor for selected thermal relaxation time and crack size are shown graphically. It is found that the stress intensity factor is considerably enhanced for large thermal relaxation time (which is a material constant) or small crack radius. In addition, the stress intensity factor at the crack front increases with the thermal relaxation time. For the case of anti-symmetric thermal flux, the mode II stress intensity factor increases rapidly with crack size. Whereas for the case of symmetric thermal flux, with increasing crack size, the mode I stress intensity factor increases slowly.

1. Introduction

The classical Fourier heat conduction law assumes that a body will be affected at the instant of heating:

$$\boldsymbol{q}(t) = -k\nabla T(t), \tag{1}$$

where t is time, $\boldsymbol{q}$ is the heat flux vector, k is the thermal conductivity and T is the temperature. This assumption means that the speed of heat propagation in the body is infinite. In practical situations, however, the speed of heat propagation in a body is always finite [1]. It has been proved that Fourier's law is not accurate in some extreme cases like under highly-varying thermal loading condition, at ultra-low temperature and in nano materials.

Cattaneo [2] and Vernottee [3] first independently formulated the non-Fourier heat conduction problem based on the local energy balance. The relaxation behavior is introduced to approach the wave nature of heat propagation and gives a non-Fourier heat conduction law:

$$\boldsymbol{q}(t+\tau_q) = -k\nabla T(t), \tag{2}$$

where τ_q is the thermal relaxation time, which is related to the collision frequency of the molecules within the energy carrier. By the non-Fourier heat conduction law and balance equation, the temperature governing equation is obtained as:

$$\rho c\tau_q \frac{\partial^2 T}{\partial t^2} + \rho c \frac{\partial T}{\partial t} = k\nabla^2 T + \tau_q \frac{\partial Q}{\partial t} + Q, \tag{3}$$

where ρ is the mass density, c is the specific heat, and Q is the internal heat generation rate per unit volume. This model is named as hyperbolic heat conduction and its accuracy have been proved by some micro/macro scope experiments [4–6].

Basically, many studies have been carried out for the classic Fourier heat conduction model (e.g., Refs. [7–10] use temperature-dependent material properties and Ref. [11] studies the heat transfer in skin tissues). Based on the work of Cattaneo and Vernotte, researchers gave several significant solutions of thermal problems. The one dimensional heat conduction in semi-infinite body [12] and wall [13] were studied early. Then the two dimensional problem in cylinder was solved [14–16]. Later, Moosaie [17–19] solved one dimensional hyperbolic heat conduction in a finite medium subjected to arbitrary periodic/non-periodic surface disturbance, and a finite medium with insulated boundaries and arbitrary initial conditions. Recently, Zhao and Wu [20] analyzed a solid sphere under sudden surface temperature change, simple harmonic periodic surface temperature change, triangular surface temperature change and pulse surface temperature changes. In addition, Babaei and Chen [21] studied the hyperbolic temperature fields of a functionally graded hollow sphere and cylinder.

* Corresponding author. Tel.: +86 075526032119.
E-mail address: guosonglin90@126.com (S.L. Guo).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.07.081
0017-9310/

It is well known that material manufacture processing usually products defects or cracks in the materials, which may disturb the local temperature distribution and intensify the temperature gradient, which introduces high thermal stress intensity that may cause rapid crack growth. Hence, the research on the mechanical behaviors of materials under thermal environments is essential [22–24]. Early, Hasselman [25] established a unified theory of thermal shock fracture initiation and crack propagation in brittle ceramics. Wilson and Yu [26] present a method of using the J-line integral to extract the magnitude of crack tip stress intensity factors from displacement solutions for thermal stress crack problems. Then Sumi and Katayama [27], and Tsai [28] investigated the thermal stress in a finite rectangular plate with a Griffith crack and thermal stress in an infinite body containing a penny-shaped crack, respectively. Lee and Shul [29] calculated the thermal stress intensity factors for an insulated interface crack in an infinite two-dimensional elastic biomaterial an under uniform heat flow. Later, Lee and Park [30] evaluated the thermal stress intensity factors for a partially insulated interface crack. The problems discussed in literatures [27–30] are under steady state thermal load. However, in practice, the transient thermal problems are significant and interesting as well. Nied and Erdogan [31] studied the thermal shock problems for a circumferentially cracked hollow cylinder. Yu and Qin [32] completed a two-dimensional analysis of thermal and electric fields of a thermopiezoelectric solid damaged by cracks based on Fourier transformations. Later, they [33] developed a generalized self-consistent approximate method for determining the thermoelectroelastic properties of piezoelectric materials weakened by microcracks. Lu and Fleck [34] solved the transient thermal problem for an orthotropic plate with an edge crack. Ma et al. [35] considered A magnetoelectrically permeable interface crack between two semi-infinite magnetoelectroelastic planes under the action of a heat flow. Guo and co-workers investigated an analytical method [36] for thermal shock crack problems of a functionally graded cylindrical shell with a circumferential crack and a combined analytical–numerical method [37] for a functionally graded plate with a surface crack, respectively. The most existing researches on the transient thermal problems are based on classical Fourier heat conduction theory. Recently, Wang and co-workers [38–42], Chen and Hu [43] and Hu and Chen [44] analyzed some thermal shock problems of infinite media, semi-infinite media, strip and plate, under the framework of hyperbolic non-Fourier heat conduction. That inspires the current consideration of the thermal shock fracture of a cylinder with an embedded penny-shaped crack.

This paper establishes a solution technique for the thermal shock fracture of cracked cylinder based on hyperbolic heat conduction. Laplace transform is used to deal with the time-varying behavior of the temperature and thermoelasticity fields. Solutions for the problem of thermally insulated crack (Section 3) and heated crack (Section 4) are obtained separately. Numerical results of stress intensity factor of the crack are evaluated to show the effects of thermal relaxation time and geometry size of the crack. Some significant differences between the classical heat conduction and the hyperbolic, non-classical heat conduction were observed.

2. Basic governing equations

Consider the axisymmetric problem of a cracked cylinder given in Fig. 1, which will be loaded with a uniform and axisymmetric axial thermal flux in the follow sections. Thus all the field variables are functions of coordinates R (radical direction) and Z (axial direction) only. The radius of the cylinder is denoted as R_b and the crack radius is denoted as R_a. The constitutive equations for the thermal

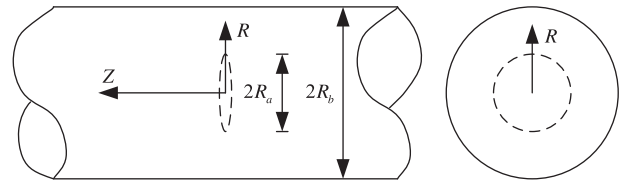

Fig. 1. A cylinder with a penny-shaped crack.

flux and the governing equation for temperature under hyperbolic heat conduction in cylindrical coordinate system are:

$$q_Z(R,Z,t) = -k\frac{\partial T(R,Z,t)}{\partial Z} - \tau_q\frac{\partial q_Z(R,Z,t)}{\partial t}, \tag{4}$$

and

$$\rho c\tau_q\frac{\partial^2 T(R,Z,t)}{\partial t^2} + \rho c\frac{\partial T(R,Z,t)}{\partial t} = k\left(\frac{\partial^2 T(R,Z,t)}{\partial R^2} + \frac{1}{R}\frac{\partial T(R,Z,t)}{\partial R} + \frac{\partial^2 T(R,Z,t)}{\partial Z^2}\right). \tag{5}$$

It has been assumed that the thermal properties are not affected by the mechanical behaviors of the material and the heat source is neglected. In order to make the study more convenient, we introduce the following dimensionless parameters according to: $\bar{t} = t/\tau_q$, $r = R/R_a$, $z = Z/R_a$, $r_b = R_b/R_a$, $a = R_a/l_0$, where $l_0 = \sqrt{k\tau_q/(\rho c)}$ is a characteristic length parameter of the material. Accordingly, Eqs. (4) and (5) can be re-written as:

$$q_z(r,z,\bar{t}) = -\frac{k}{R_a}\frac{\partial T(r,z,\bar{t})}{\partial z} - \frac{\partial q_z(r,z,\bar{t})}{\partial \bar{t}}, \tag{6}$$

and

$$\frac{\partial^2 T(r,z,\bar{t})}{\partial r^2} + \frac{1}{r}\frac{\partial T(r,z,\bar{t})}{\partial r} + \frac{\partial^2 T(r,z,\bar{t})}{\partial z^2} = a^2\left(\frac{\partial^2 T(r,z,\bar{t})}{\partial \bar{t}^2} + \frac{\partial T(r,z,\bar{t})}{\partial \bar{t}}\right). \tag{7}$$

Both boundary and initial conditions are necessary for the solution of Eq. (7). Two cases of boundary conditions will be considered in Sections 3 and 4 separately. The initial conditions $T(r,z,0) = T_0$ and $q_r(r,z,0) = q_z(r,z,0) = 0$ can simulate most practical cases. However, the thermal stress of interest is associated with temperature change $\Delta T = T(r,z,\bar{t}) - T_0$. To simplify the deducing of solutions for mechanical fields, $T_0 = 0$ is assumed, which means that $\Delta T = T(r,z,\bar{t})$. This paper does not consider the temperature dependency of material properties. Thus, assumption of $T_0 = 0$ does not affect the thermal stress level. Now, the initial conditions of Eq. (7) are:

$$T(r,z,0) = 0, \quad \partial T(r,z,0)/\partial r = \partial T(r,z,0)/\partial z = 0 \tag{8}$$

In the following analysis, Laplace transform will be applied to Eqs. (6) and (7). Then the temperature field in Laplace transform will be obtained, and be used for solving the thermal stresses.

Base on the linear thermoelastic theory, the mechanical constitutive equations are:

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \\ \sigma_{rz} \end{Bmatrix} = \frac{1}{R_a}\begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 \\ c_{12} & c_{11} & c_{13} & 0 \\ c_{13} & c_{13} & c_{33} & 0 \\ 0 & 0 & 0 & c_{44} \end{bmatrix}\begin{Bmatrix} \partial u/\partial r \\ u/r \\ \partial w/\partial z \\ \partial u/\partial z + \partial w/\partial r \end{Bmatrix} - \begin{Bmatrix} \chi_{11} \\ \chi_{11} \\ \chi_{33} \\ 0 \end{Bmatrix} T, \tag{9}$$