International Journal of Heat and Mass Transfer 91 (2015) 255–263

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

An analytical permeability model for power-law fluids in porous fibrous media with consideration of electric double layer

Q.Y. Zhu [a], Y.J. Zhuang [a], H.Z. Yu [a,b,*]

[a] School of Engineering, Sun Yat-sen University, Guangzhou 510275, China
[b] China Earthquake Networks Center, Beijing 100045, China

ARTICLE INFO

Article history:
Received 27 March 2015
Received in revised form 28 July 2015
Accepted 30 July 2015
Available online 8 August 2015

Keywords:
Permeability
Power-law fluids
Porous fibrous media
Fractal model
Electric double layer (EDL)

ABSTRACT

This work studies the permeability of power-law fluids in porous fibrous media with electrokinetic effects. By solving the linearized Poisson–Boltzmann and Navier–Stokes equations, we get the analytical solutions of pressure driven flow of power-law fluids in a microcapillary with electric double layer (EDL). The flow rate in a single capillary, combining with the fractal model of pore distribution of fractures in naturally fractured porous media, deduces the total flow rate. Then the analytical result of effective permeability for power-law fluids with EDL effects is derived as a function of the porosity, the flow behavior index and a dimensionless number derived from the solid surface zeta potential and maximum pore radius. The present results show that the EDL effects as well as other variable parameters may greatly influence the effective permeability of the power-law fluids in porous fibrous media: the larger the porosity, the higher the effective permeability; the larger the maximum pore radius, the higher the effective permeability; the higher the solid surface zeta potential, the more the EDL effects; the more the EDL effects, the lower the effective permeability. Comparing the effective permeability produced by different flow behavior indexes, we further illustrate that the EDL has virtually no effects when the flow behavior index is great than 1, moderate effects when equal to 1, and very significant effects when less than 1. Therefore the EDL effects may provide strong constraints on evaluation of the effective permeability of the shear thinning fluids rather than the shear thickening and Newtonian fluids.

1. Introduction

Porous media has been broadly applied to heat exchangers, medical science, and other aspects [1,2]. One of the most important measures of characterizing the transport properties in porous media is the permeability. Analytical expressions for calculating permeability of the porous media are the goal of many researchers in the field of the flow through porous media [3,4]. For examples, a Fast Fourier Transform (FFT) based method was presented to compute the dynamic permeability of periodic porous media by Nguyen [5]. The Monte Carlo technique was proposed to develop a probability model for radial permeability in fractured porous media [6].

In fact, the determination of the permeability depends strongly on the porosity and pore radius as well as geometrical formation factors [7,8]. Due to the random distribution of the fibers in porous fibrous media, the pores in porous media are non-uniform in size; causing a great difference between the traditional models and the real cases exists. Fortunately, many researchers have found that the pore size distribution in porous media can be described well by using the fractal geometry [9,10], promoting the studies of this aspect. Xu [11] used the fractal geometry theory to investigate the permeability of the fractal-like tree network by parallel and series models. Zhu [12] developed a fractal model to analyze the effects of porosity of porous fibrous media on heat and mass transfer. Miao [13] proposed a fractal model to analyze the permeability of the rocks with shear fractures which widely exist in nature such as oil/gas reservoirs.

However, for the analysis of the pressure driven flow in porous fibrous media, the electrokinetic effects should be included. Considerable researches of the electroviscous effects (EDL effects) on microchannel flow have been reported in recent years. Zhao [14] presented a comprehensive review of electrokinetics flow of non-Newtonian fluids. Power-law non-Newtonian flow in microchannels combined with electroviscous effect was numerically simulated by Tang [15]. Vakili [16] also studied the electroosmotic flow of power-law fluids in rectangular microchannels by using the numerical method. Quite recently, Zhu [17] provided a fourth-order compact difference method to discuss the periodical

---

* Corresponding author at: School of Engineering, Sun Yat-sen University, Guangzhou 510275, China.

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.07.127
0017-9310/

Nomenclature

A_0	cross-sectional flow area (m^2)
A	the total cross-sectional area of a unit cell (m^2)
D_F	fractal dimension
d	Euclidean dimension
E_z	electric field vector (V/m)
$\bar{E}_z$	dimensionless electric field
e	the elementary charge (C)
$\mathbf{F}$	body force vector
$\bar{G}_1$	a dimensionless parameter
$\bar{G}_2$	a dimensionless parameter
J_c	conducting current density (A/m^2)
J_s	streaming current density (A/m^2)
K	permeability tensor of the porous materials
k	dimensionless electroosmotic radius
k_B	Boltzmann constant
L	length scale
m	dynamic viscosity of power-law fluids ($N\,m^{-2}\,s^n$)
N	number of pores or capillaries in material
N_t	total number of pores or capillaries in material
n_∞	the bulk ionic concentration (m^{-3})
n	flow behavior index
p	pressure in the capillary (N/m^2)
$\bar{P}_z$	dimensionless pressure gradient
Q	total flow rate of liquid moisture through the material (m^3/s)
q	flow rate of liquid moisture thought a single microchannel (m^3/s)
R_c	mean pore radius (m)
R_f	radius of fibers (m)
R_0	capillary radius in material (m)
R	radial coordinate of a material (m)
$\bar{R}$	dimensionless radial coordinate of a material
$R_{\min}$	minimum pore radius in material (m)
$R_{\max}$	maximum pore radius in material (m)

T	absolute temperature (K)
U	volume-averaged superficial fluid velocity (m/s)
$\mathbf{V}$	flow velocity vector
V	maximum velocity of capillary without consideration of EDL
$\bar{v}(\bar{R})$	dimensionless velocity
v_z	velocity of the fluid (m/s)
$v(\bar{R})_{mean}$	mean velocity of the liquid (m/s)
z	z-coordinate

Greek symbols

θ	contact angle
ψ	local electrical potential (V)
Ψ	dimensionless local electrical potential
ψ_w	wall zeta potential (V)
ρ	mass density ($kg\,m^{-3}$)
ρ_e	the net charge density per unit volume (C/m^3)
ε	the relative dielectric permittivity
ε_0	the permittivity of vacuum ($C\,V^{-1}\,m^{-1}$)
κ	Debye–Huckel parameter (m^{-1})
χ	valence number of the ion
ζ_0	zeta potential (V)
λ	electrical conductivity of the electrolyte solution (S/m)
ϕ	porosity of the fibrous materials
ϕ_i	micro-porosity
ϕ_{eff}	effective porosity
$\mathbf{\Theta}$	dimensionless velocity variable defined in Eq. (28)
$\dot{\boldsymbol{\gamma}}$	strain rate tensor
$\dot{\gamma}$	the magnitude of strain rate tensor
$\dot{\gamma}_w$	shear rate for power-law fluids in a single capillary
$\boldsymbol{\tau}$	stress tensor
μ	the apparent viscosity for power-law fluids
ω	a modified coefficient
δ	ratio of $R_{\min}$ to $R_{\max}$

flow of power-law fluids through a rectangular microchannel with electroviscous effects. These results indicate that the EDL effect can be an important factor to assess the pressure driven flow in porous media [18–20]. Nowadays, the electrokinetic phenomena in porous media have widely been applied to the micropump, chemical engineering, medical science and other fields [21,22].

Above studies of fluid flow in porous media are focused on the Newtonian fluids. Actually, complex fluids (e.g. polymer solutions and colloids) with long-chain molecules exhibiting obvious non-Newtonian characteristics (e.g. changes in viscosity, memory effect, yield stress and hysteresis fluid properties) are also manipulated in medical and hygiene applications (e.g. blood). Many studies basically follow the works in this regard. Chen [23] used the lattice Poisson–Boltzmann method (LPBM) to explore the effects of the porous medium structure on electro-osmotic permeability of a power-law fluid. Turcio [24] applied the Bautista–Manero–Puig (BMP) model to analyze the effective permeability in fractal porous media. The power-law fluids were numerically simulated in 3-D fibrous structures to study the effects of fibers orientations on the permeability of fibrous media by Emami [25]. Presently, however, the analytical study related to the EDL effects on permeability of power-law fluids in porous fibrous media has not been reported. And, few analytical studies, which can be incorporated into the fractal model of pore distribution in naturally fractured porous media, have been carried out to investigate the power-law fluids flow in a cylindrical microcapillary with electrokinetic phenomena.

In this paper, we experiment an analytical model, by using the fractal technique of pore distribution in porous fibrous media, to investigate the effective permeability of power-law fluids with EDL effects. We investigate the variations of the effective permeability with the porosity, solid surface zeta potential, and maximum pore radius. Meanwhile, we study the effects of EDL on the evaluation of the effective permeability. To show the validity of the model, the experimental data given by Kostornov [26] and Labrecque [27] are tested. In addition, the relationship between the EDL effect and the flow behavior index is also discussed.

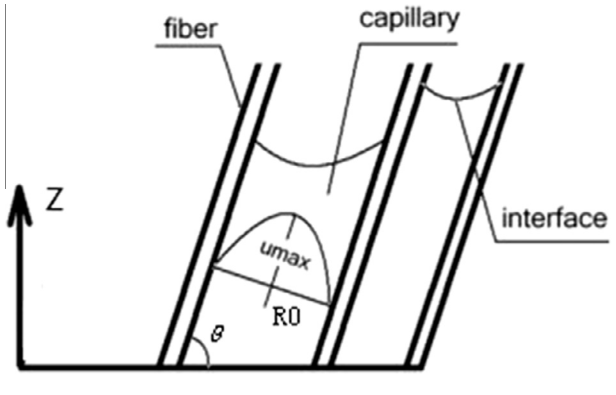

Fig. 1. Schematic diagram of the capillaries. R_0 denotes radius of the capillary.