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Combined effects of fluid shear-thinning and yield stress on heat transfer from an isothermal spheroid

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ABSTRACT

In this work, the effect of aspect ratio (polar to equatorial axis) of a spheroid on the flow and heat transfer in shear-thinning viscoplastic fluids characterised by the Herschel–Bulkley fluid model has been analyzed in the forced-convection regime. The momentum and energy equations have been solved numerically in the steady and laminar flow regime over the following ranges of conditions: Reynolds number, $1 \leqslant Re \leqslant 100$; Prandtl number, $1 \leqslant Pr \leqslant 100$; Bingham number, $0 \leqslant Bn \leqslant 10$; power-law index, $0.2 \leqslant n \leqslant 1$ and the aspect ratio of the spheroid, $0.2 \leqslant e \leqslant 5$. In addition, limited results were also obtained in the low Reynolds number ($Re \to 0$) and Peclet number regime to examine the scaling of the Nusselt number with the Peclet number. The effect of particle shape is elucidated on the size and location of yield surfaces, streamline and isotherm contours, wake characteristics (length and separation angle), drag coefficient and the local and average Nusselt numbers over the foregoing ranges of conditions. In general, oblate shapes ($e < 1$) promote heat transfer with reference to that for a sphere ($e = 1$) at fixed values of the Reynolds, Prandtl and Bingham numbers. The tendency for wake formation is, however, reduced by the fluid yield stress. All else being equal, both drag and Nusselt number show a positive dependence on the Bingham number due to the sharpening of the gradients in the thin fluid-like regions existing adjacent to the spheroid. Further augmentation in heat transfer is achieved by introducing shear-thinning fluid behaviour in yield-stress fluids. The paper is concluded by presenting a correlation in terms of the Colburn-j factor as a function of the modified Reynolds number (Re^*), power-law index (n) and aspect ratio (e) thereby enabling the estimation of the Nusselt number for intermediate values of parameters in a new application.

1. Introduction

Most structured fluids like emulsions, foams, dispersions and suspensions, melts, solutions and composites made of high molecular weight natural and synthetic polymers and micellar solutions, etc. exhibit a range of non-Newtonian flow characteristics which, in turn, give rise to a range of spectacular flow phenomena [1,2] as well as pose enormous challenges to their processing in scores of industrial settings [3–6]. The most common and frequently encountered non-Newtonian characteristics are the so-called shear-thinning viscosity and fluid yield stress. From an engineering standpoint, it is now widely acknowledged that the fluid yield stress not only makes their flow difficult but heating of such fluids in heat exchangers and other devices is also severely impeded [2,7]. This trend is also borne out by some of our recent studies on the laminar natural- and forced-convection heat transfer from heated spheres [8,9], spheroids [10], circular and elliptical cylinders [11–14] in Bingham plastic fluids. In contrast, the shear-thinning fluid behaviour facilitates heat transfer over and above that seen in Newtonian fluids. Indeed, it is possible to enhance the rates of heat transfer by up to 70–80% under appropriate conditions. Such augmentation in heat transfer has been demonstrated for a sphere [15–20], and circular [21,22] and elliptical cylinders [23,24] in the forced-, free- and aiding-buoyancy mixed convection regimes. Of course, the degree of enhancement in heat transfer varies from one shape to another and is strongly dependent on the values of the shear-thinning index and the values of the influencing parameters like Reynolds and Prandtl numbers in the forced-convection and Grashof and Prandtl numbers in the free-convection regime, etc. Broadly, stronger the shear-thinning behaviour and advection, greater is the enhancement in heat transfer. It thus stands to reason that the reduction in heat transfer on account of the fluid yield stress can be partially compensated if the fluid also exhibits shear-thinning behaviour. Indeed, most viscoplastic fluids do exhibit varying levels of shear-thinning

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Nomenclature

a	spheroid semi-axis normal to flow (equatorial semi-axis), m
A_p	projected area of spheroid normal to flow ($=\pi a^2$), m^2
b	spheroid semi-axis parallel to flow (polar semi-axis), m
Bn	Bingham number $\left(=\frac{\tau_0}{K(U_0/2a)^n}\right)$, dimensionless
C_D	drag coefficient $\left(=\frac{2F_D}{\rho U_0^2 \cdot A_p}\right)$, dimensionless
C_{DF}	friction drag coefficient $\left(=\frac{2F_{DF}}{\rho U_0^2 \cdot A_p}\right)$, dimensionless
C_{DP}	pressure or form drag coefficient $\left(=\frac{2F_{DP}}{\rho U_0^2 \cdot A_p}\right)$, dimensionless
c_p	specific heat of fluid, J kg^{-1} K^{-1}
C_p	pressure coefficient
D_∞	diameter of computational domain, m
e	aspect ratio of spheroid ($=b/a$), dimensionless
F_D	total drag force, N
F_{DF}	friction component of drag force, N
F_{DP}	pressure component of drag force, N
F_s	Stokes drag force for a sphere, N
j	Colburn-j factor, dimensionless
k	thermal conductivity of fluid, W m^{-1} K^{-1}
K	fluid consistency index in Herschel–Bulkley model, Pa s^n
L_r	recirculation length measured from the spheroid surface in the direction of flow, m
m	regularization parameter in Papanastasiou approximation, Eq. (7), dimensionless
n	power-law index, dimensionless
N_p	number of grid points on the surface of spheroid, dimensionless
Nu	average Nusselt number, dimensionless
Nu_θ	local Nusselt number on the surface of spheroid, dimensionless
Nu_∞	average Nusselt number in the limit of conduction, dimensionless
P	pressure, dimensionless
Pe	Peclet number, dimensionless
Pr	Prandtl number $\left(=\frac{K c_p}{k}\left(\frac{U_0}{2a}\right)^{n-1}\right)$, dimensionless
Pr^*	modified Prandtl number($=Pr(1+Bn)$), dimensionless
Re	Reynolds number$\left(=\frac{(2a)^n U_0^{2-n}\rho}{K}\right)$, dimensionless
Re_{2b}	Reynolds number based on the length scale of $2b$, dimensionless
Re^*	modified Reynolds number$\left(=\frac{Re}{(1+Bn)}\right)$, dimensionless
T	fluid temperature, K
T_0	temperature of the fluid in the free stream, K
T_w	temperature on the surface of spheroid, K
U_0	free stream velocity, m s^{-1}
$\mathbf{V}$	velocity vector, dimensionless
x, y	Cartesian coordinates, m

Greek symbols

$\dot{\gamma}$	rate of deformation tensor, dimensionless
δ	minimum grid spacing on the surface of spheroid, m
η	apparent viscosity of fluid, dimensionless
θ	position on the surface of spheroid, degree
θ_s	separation angle, degree
λ	parameter used in Bercovier and Engelman model, dimensionless
μ_{HB}	plastic viscosity of Herschel–Bulkley fluid model ($=K(U_0/2a)^{n-1}$), Pa s
μ_y	yielding viscosity in the bi-viscous model, Pa s
ξ	fluid temperature $\left(=\frac{T-T_0}{T_w-T_0}\right)$, dimensionless
ρ	density of fluid, kg m^{-3}
$\boldsymbol{\tau}$	deviatoric stress tensor, dimensionless
τ_0	fluid yield stress, Pa

behaviour [6,25,26]. This conjecture is in line with the scant results on forced convection heat transfer from a sphere in Herschel–Bulkley model fluids [27]. The present work aims to examine this proposition by considering forced convection heat transfer from an isothermal spheroidal particle to Herschel–Bulkley model fluids in the steady axisymmetric flow regime. It is, however, instructive to tersely review the pertinent studies in Newtonian fluids available in the literature which, in turn, will facilitate the presentation and discussion of the new results.

In contrast to the voluminous literature for a sphere [28,29] and the continued interest in this problem [30], the analogous body of information on spheroids is very limited even in Newtonian fluids. Early studies of Payne and Pell [31] and Breach [32] have dealt with the calculation of drag on axisymmetric bodies including prolates and oblates in the creeping flow regime. Subsequently, these have been supplemented by the numerical studies at finite Reynolds numbers for steady and time-dependent flows [33–39] and see the references therein. All in all, the numerical results are now available up to Reynolds number values of ~100 or so depending upon the linear dimension used. For instance, Alassar and Badr [37–39] have numerically investigated the steady [37] and time-dependent (both starting from rest and pulsating) [38,39] flow of Newtonian fluids over a spheroid, particularly to delineate the effect of Reynolds number (0.1–1) on the flow field in the steady flow regime. The time-dependent studies endeavour to elucidate the effect of amplitude and frequency of pulsations on the evolution of pressure and velocity fields and drag coefficients, etc. Aside from these numerical studies, some results have also been obtained on forced convection heat transfer from particles of arbitrary and/or slender shapes under limiting conditions [40,41]. Thus, for instance, Brenner [40] considered forced convection heat transfer from a particle of arbitrary shape in the Stokes flow regime for small values of Peclet number. He used the standard approach of matching the so-called inner and outer expansions for the temperature fields. These results were expressed in terms of the ratio (Nu/Nu_∞), i.e., augmentation in heat transfer above the conduction limit. Interestingly, this ratio was found to be independent of the Reynolds number up to the first order in Peclet number whereas higher order terms depended on both the Reynolds number as well as the orientation of the particle. In a recent study [41], Schnitzer has provided an excellent overview of the extension of this approach in the context of advection–diffusion of heat and mass by combining it with the slender-body approximation. He has considered the case of arbitrary tangential velocity distribution on the surface of the slender body of revolution including the limiting cases of irrotational and no-slip Stokes flow. Of particular interest are his results on the forced convection heat transfer from an isothermal slender body (aspect ratio, $e \to \infty$) which are valid up to Peclet numbers of order 1 in the Stokes flow regime. In the limit of small Peclet numbers, his predictions are consistent with that of Brenner [40]. Furthermore, his calculations show the error to be logarithmically small in e. Lastly, the slender-body approach outlined in [41] does not seem to be limited to small values of Peclet number as is evident from a close match between the analytical and numerical results up to about $Pe \leqslant 7$. Similarly, Dwyer and Dandy [42] reported numerical results in the range $10 \leqslant Re \leqslant 66$