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Azimuthal instability modes in a viscoelastic liquid layer flowing down a heated cylinder

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ABSTRACT

In this paper the non axisymmetric longwave instability of a thin viscoelastic liquid film flowing down a vertical heated cylinder is investigated. The stability of the film coating a cylinder in the absence of gravity is also investigated. In a previous paper it is found that viscoelasticity stimulates the appearance of azimuthal modes but the axial mode is the most unstable one. Other calculations in a former paper show that for flow outside a heated cylinder azimuthal modes can be the more unstable when the Marangoni number is large and, in particular, when the Reynolds number and wavenumber are small. Therefore, the small wavenumber and large cylinder radius approximation is assumed with the simultaneous action of viscoelasticity and thermocapillarity on the stability of azimuthal modes. In the presence and in the absence of gravity, it is found that, in comparison with the Newtonian case, it is easier to excite the azimuthal modes when viscoelasticity and thermocapillarity destabilize at the same time. Moreover, it is shown that, despite the axial mode is the most unstable one, there are wide wavenumber ranges where higher modes are the more unstable and they can show up by means of a periodic time dependent perturbation.

1. Introduction

The coating of surfaces by liquid films have important applications in industry. The problems found when looking for the perfect finishing are due to hydrodynamic instabilities. In the absence of gravity a cause of instability is thermocapillarity. When the liquid layer is coating a flat wall Pearson [1] has shown that a liquid film is unstable to temperature gradients perpendicular to the layer. As a consequence convection cells appear which may have important consequences in the solidified film. Therefore, it is necessary to investigate this instability under different mechanical and thermal boundary conditions. When the free surface is deformable the problem is investigated first by Scriven and Sternling [2]. The restoring influence of gravity is taken into account by Takashima [3] in the stationary case and by Takashima [4] when the flow is time dependent. The double diffusive Marangoni convection is first investigated by Mctaggart [5]. Sometimes in applications the fluid has elastic properties due to the presence in solution of macromolecules which change their form when shear stresses are applied to the liquid. These fluids are called viscoelastic (see for example Bird et al. [6]) and have been investigated widely in natural convection phenomena (see a recent review paper by Dávalos-Orozco [7–9] by Pérez-Reyes and Dávalos-Orozco). Notice that one characteristic of the viscoelastic instabilities is that they can be time dependent, in contrast to Newtonian fluids convection. Yet it is shown [8] that these instabilities do not occur for any thermal boundary conditions.

The thermal Marangoni instability has also been investigated for viscoelastic fluids by a number of authors. Getachew and Rosenblat [10] calculated the codimension-two points where stationary and oscillatory convection compete to be the first unstable one when the Marangoni number increases. Wilson [11] investigates supercritical conditions of the thermocapillary instability of a viscoelastic fluid from the point of view of the growth rates. Siddheshwar et al. [12] investigate the instability of a Maxwell fluid under different thermal boundary conditions including the effect of viscosity variation with temperature. The thermocapillary instability of a Maxwell viscoelastic fluid is investigated by Hernández-Hernández and Dávalos-Orozco [13] assuming a flat free surface and presenting results for a wide range of wall thermal conductivities. The goal is to calculate the codimension-two points where the stationary and oscillatory Marangoni convection modes compete to be the first unstable one.

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Nomenclature

Bi	free surface-atmosphere Biot number
c	phase velocity
Cr	crispation number
De	Deborah number
$\mathbf{e}$	shear rate tensor
g	acceleration of gravity
h	free surface deformation
h_0	mean thickness of the layer
H_h	heat transfer coefficient
H	free surface perturbation amplitude
k	axial wavenumber
k_C	critical wavenumber
k_f	fluid thermal conductivity
L_1	adimensional relaxation time
L_2	adimensional retardation time
m	azimuthal number
Ma	Marangoni number
$\vec{n}$	normal vector
P	pressure
p_i : i-th	order perturbation pressure
Pr	Prandtl number
R	cylinder radius
Re	reynolds number
S	surface tension number
$\bar{S}$	scaled surface tension number
$\bar{T}$	temperature
T_{amb}	ambient temperature
T_i : i-th	order perturbation temperature
T_W	wall temperature
U	representative velocity
$\vec{V}$	velocity vector
We	Weber number

Greek

α	fluid thermal diffusivity
β	non dimensional cylinder radius
γ	surface tension
δ	scaled non dimensional cylinder radius
ΔT	temperature difference
ρ	fluid density
ν	kinematic viscosity
σ	growth rate
τ	shear stress tensor
$\vec{\tau_1}$	first tangential vector
$\vec{\tau_2}$	second tangential vector
ω	frequency of oscillation

When a fluid layer flows down a wall, the thermocapillary effects are included by Joo et al. [16,14] and Ramaswamy et al. [15]. A complete review of this problem is found in Dávalos-orozco [17].

Nonlinear computations of the instability of a thin viscoelastic film falling down an inclined wall are done by Joo [18]. Kang and Chen [19] find in the linear limit a purely elastic instability. This flow is investigated by Dávalos-Orozco [20] when the wall is smoothly deformed. It is shown that it is still possible to stabilize the flow by means of spatial resonance as done by Dávalos-Orozco [21] when the fluid is Newtonian.

It is of interest to know if the azimuthal modes are relevant in a cylindrical wall. Shlang and Sivashinsky [22] found that the azimuthal modes can not be the most unstable in a Newtonian fluid and that the axial one is always the most unstable one. For flow inside the cylinder the axial mode grows faster as in microchannels when the liquid forms an annular film [23,24]. Therefore, for any radius, the axial mode is the most unstable one inside the cylinder. When a film is flowing down the outside of a rotating cylinder, it has been shown [25–27] that the first azimuthal mode may be the most unstable one under different circumstances. Nevertheless, for flow inside the cylinder (as in [28]) the most unstable mode is the axial one. The relevance of the azimuthal modes is also found in the instability of inviscid stratified fluids in a rotating annulus [29].

The thermocapillary phenomena of a film flowing down a vertical cylinder present interesting results. This free surface condition is of concern in practical applications of heat dissipation [30]. Linear stability calculations of a thin film flowing down a cylindrical heated wall (see Dávalos-Orozco and You [31]) have demonstrated that high azimuthal modes can be the more unstable ones when the Reynolds number and the wavenumber of the perturbation are small. To excite these modes large magnitudes of the temperature gradient are required. It is important to point out that in the presence of thermocapillarity, the azimuthal modes can also be excited as the more unstable ones when the flow is inside the cylinder.

The two dimensional flow instability of non-Newtonian thin films flowing down a cylinder has also been investigated by Cheng and Liu [32–34] for a power-law fluid, by Cheng et al. [35] and Cheng and Lai [36] for a viscoelastic Walters B fluid (with application to magnetohydrodynamics). In Moctezuma-Sánchez and Dávalos-Orozco [37] the viscoelastic Oldroyd's constitutive equation model was used to investigate the longwave linear instability of a fluid film flowing down a cylinder. The corresponding linear equation reduces to that obtained by Joo [18] (without power-law fluid effects) when the radius of the cylinder tends to infinity. In particular, the interest in [37] is to determine the relevance of the azimuthal modes in the presence of viscoelasticity. It is found that the most unstable mode is always the axial one. Eventhough, viscoelasticity promotes the appearance of the azimuthal modes in comparison with the Newtonian fluid, they are not the more unstable ones in any range of the wavenumber.

In the present paper, the interest is focused on the thermocapillary excitation of azimuthal modes in a viscoelastic fluid. A comparison is done with the results of the isothermal [37] flow and the Newtonian fluid [31] flow. The Oldroyd's fluid model is selected for the constitutive equation of the fluid. The linear evolution equation calculated below, reduces to that of Joo [18] when the radius of the cylinder tends to infinity and in the absence of thermocapillary effects. In the lack of thermocapillary effects the equation reduces to that in [37]. The results of this paper are new not only because of the combination of viscoelasticity [37] and thermocapillarity [31] in flow on the surface of a cylinder, but also because the problem investigated is three dimensional. This can be seen in the review section on thin film flow down cylinders presented in Dávalos-orozco [17]. It is found that in the linear and non linear problems, mainly axial mode stability is investigated. For three dimensional flows see [22,25–27,29]. The physical reason for the appearance of azimuthal modes of instability are the azimuthal shear stresses created by thermocapillarity, as will be seen presently in the discussion of the first and second tangential shear stresses of the free surface boundary conditions.