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# Non-continuum effects on natural convection–radiation boundary layer flow from a heated vertical plate

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ABSTRACT

Heat transfer by simultaneous radiation and natural convection through an optically thick fluid over a heated vertical plate has been studied with first-order momentum and thermal non-continuum boundary conditions. The radiant heat flux was treated using the Rosseland diffusion approximation. By solving the local non-similarity two-equation model, numerical solutions were obtained to examine the slip effects on the interaction between radiation and natural convection for a range of rarefied conditions and radiation effects. Results including slip velocity, temperature jump, skin friction, and heat transfer rate are presented graphically and discussed. In addition, an integral correlation is presented for the average Nusselt number as a function of the non-continuum conditions, radiation–conduction parameter, and flow properties.



## 1. Introduction

Thermal radiation effects on natural convection flow are important in the context of space technology and applications involving high temperatures. Examples of combined radiation and natural convection problems are found in a wide variety of engineering applications, including high temperature material processing such as oxide crystal melt flow and glass production, high temperature heat exchangers, and so forth [1]. The interaction between radiation and natural convection has been studied by many researchers in the past with a major focus on the configuration of semi-infinite flat plates. Cess [2] studied the natural convection of an absorbing-emitting fluid along a vertical flat plate, assuming a relatively small importance of conduction compared to radiation. Arpaci [3] qualitatively analyzed the interaction between radiation and natural convection from a heated vertical plate, obtaining integral forms of radiant flux for both thin and thick gases. Ali et al. [4] investigated the radiation effects on the boundary-layer flow of a gray medium using the Rosseland approximation. Results suggested that radiation enhances shear stress and surface heat transfer rate. Hossain and Takhar [5] and Hossain and Alim [6] examined the radiation effects on natural convection past an isothermal horizontal plate and a thin vertical cylinder, respectively. The results indicated that key flow and heat transfer characteristics are altered within the boundary layer by interacting with radiation. Chamkha et al. [7] considered the impact of mass transfer studying the radiation–natural convection interaction, and transformed the boundary-layer equations to non-similar forms. Employing the normal-mode-expansion technique, Cheng and Ozisik [8] showed that the radiation effects enhance heat transfer and boundary layer thickness. Using the perturbation method, Martynenko et al. [9] investigated the radiation–convection interaction in nonparticipating air over a vertical surface with a uniform heat flux. Numerical results were shown to agree well with known experimental data. Webb [10] conducted experiments to validate the numerical results obtained for buoyance-driven flow coupled by radiation boundary conditions. In addition to the research done on semi-infinite flat plates, the interaction between natural convection and radiation was also studied with other geometry configurations, such as square enclosures by Yucel et al. [11], and Akiyama and Chong [12], and vertical channels by Sparrow et al. [13].

The majority of previous studies have focused on the continuous flow regime. With the increasing interest in micro-scale heat transfer in advanced industrial applications, simultaneous natural convection and radiation heat transfer in slip regime is worth in-depth understanding. As the mean free path of the flow approaches the characteristic length scale of the problem, flows will demonstrate non-continuum behavior due to fewer molecular collisions within the dimension of interest. The degree of the rarefaction for gaseous flows is usually characterized by Knudsen

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**Nomenclature**

| | |
|---|---|
| $C_f$ | skin friction coefficient |
| $c_p$ | specific heat at constant pressure |
| $f$ | momentum accommodation coefficient |
| $F$ | transformed stream function |
| $G$ | $\xi$-derivative of $F$ |
| $g$ | gravitational acceleration |
| $Gr$ | Grashof number, $Gr = g\,\beta\Delta TL^3/\nu^2$ |
| $h$ | heat transfer coefficient |
| $k$ | thermal conductivity |
| $Kn$ | Knudsen number, $Kn = \lambda/L$ |
| $L$ | characteristic length |
| $Nu$ | Nusselt number, $Nu = hL/k$ |
| $p$ | pressure |
| $Pr$ | Prandtl number, $Pr = \mu\,c_p/k$ |
| $q$ | heat flux |
| $T$ | temperature |
| $u$ | streamwise velocity |
| $v$ | normal velocity |
| $x$ | coordinate along the plate |
| $y$ | coordinate normal to the plate |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | thermal accommodation coefficient |
| $\beta$ | volumetric thermal expansion coefficient |
| $\gamma$ | specific heat ratio of air |
| $\eta$ | pseudo-similarity variable |
| $\theta$ | dimensionless temperature |
| $\xi$ | dimensionless non-continuum variable |
| $\lambda$ | mean free path |
| $\mu$ | dynamic viscosity |
| $\nu$ | kinematic viscosity |
| $\tau$ | shear stress |
| $\varphi$ | $\xi$-derivative of $\theta$ |
| $\Psi$ | stream function |

*Subscripts*

| | |
|---|---|
| $L$ | length of entire plate |
| $r$ | radiation |
| $x$ | local $\times$ position |
| $slip$ | slip condition |
| $w$ | wall boundary |
| $\infty$ | ambient conditions |

*Superscripts*

| | |
|---|---|
| $tt$ | total |
| $conv$ | convection component |
| $rad$ | radiation component |

number defined as the ratio of molecular mean free path to the characteristic length of the problem of interest, given by [14]

$$Kn = \lambda/L \tag{1}$$

Based on the Knudsen number value, flows can be classified into three regimes: continuum flow ($Kn < 0.01$), slip flow ($0.01 \leqslant Kn \leqslant 0.1$), and transitional flow ($0.1 < Kn < 10$) [14]. As the flow approaches the continuum limit, the conventional no-slip wall boundary condition fails to model the surface interaction between the fluid and the wall boundary due to the low collision frequency [14]. Slip models were proposed to improve the prediction of the non-continuum phenomenon near wall boundaries within the framework of the continuum assumption. For gaseous flows, Maxwell slip model relates the slip velocity at the wall to the local velocity gradient based on the gas kinetic theory, given by [15]

$$u_{slip} \approx \left(\frac{2}{f} - 1\right)\lambda \frac{du}{dy}|_w \tag{2}$$

In addition, as the Knudsen number exceeds the continuum limit, temperature jump may exist between the gas molecules and the wall boundary. The interfacial temperature discontinuity physically accounts for gas molecules not thermally accommodated with the wall and thereby is critical to the prediction of energy transfer. Analogous to the model of velocity slip, the kinetic theory expression for first-order temperature discontinuity is given by [15]

$$T_{slip} - T_w \approx \left(\frac{2}{\alpha} - 1\right)\frac{2\gamma}{\gamma + 1}\frac{\lambda}{Pr}\frac{dT}{dy}|_w \tag{3}$$

Recent studies of non-continuum effects on external boundary layer flow have shown that such interfacial discontinuities have substantial impact on altering the structure of boundary layer and heat transfer behavior. Martin and Boyd [16] investigated the forced convection over a horizontal plate with slip conditions using finite difference methods. Results suggested that the flow structure, velocity profile, and boundary layer thickness are changed by the non-continuum conditions. Cao and Baker [17,18] studied the first-order slip effects upon natural and mixed convection from a vertical plate, showing that velocity and temperature profiles as well as heat transfer characteristics are significantly altered as a result of the presence of slip conditions. Merkin and Pop [19], and Aman et al. [20] numerically investigated the slip effects on mixed convection for both the aiding and opposing cases. In addition, a number of recent studies examined slip effects for boundary layer flows over various geometric configurations including horizontal plate, cylinder, and disc can be found in the literature [21–23]. The effects of simultaneous momentum and thermal slip upon the radiation and natural convection boundary layer flow have not received much attention, though the interfacial discontinuity is important to predicting the flow and heat transfer behavior at slightly rarefied conditions. The only slip flow studies including radiation and natural convection on a vertical plate presented so far were in a porous medium [30–31].

The purpose of the present paper is to provide a better understanding of how the interfacial discontinuity impacts the radiation and natural convection boundary layer flow. To study the mechanism, a number of simplifying assumptions, such as a gray fluid and Rosseland diffusion approximation, are employed. This paper provides local non-similar solutions to the boundary-layer equations for radiation and natural convection flow over a vertical heated plate subject to first-order slip boundary conditions. Due to the simplifying Rosseland approximation, the results presented in this paper applies mostly to optically thick gases that emit and absorb but do not scatter radiation.

## 2. Natural convection–radiation flow with slip

Consider a natural convection through an optically dense fluid over a vertical flat plate as shown in Fig. 1. The plate is maintained at a uniform surface temperature $T_w$ ($T_w > T_\infty$). The $x$ ordinate is measured along the vertical plate from the leading edge, whereas $y$ ordinate normal to the plate.