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Fuzzy stochastic finite element method for the hybrid uncertain temperature field prediction

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ABSTRACT

For the hybrid uncertain temperature field prediction involving both random and fuzzy uncertainties in material properties, external loads and boundary conditions, this paper proposes a new numerical technique named fuzzy stochastic finite element method (FSFEM) by a combination of perturbation theory and moment method. Random variables are adopted to quantify the stochastic uncertainty with sufficient experiment data; whereas, fuzzy variables are used to represent the non-probabilistic parameters associated with expert opinions. By using the level-cut method, the fuzzy parameters are equivalently decomposed into interval variables. Based on the first-order Neumann series and random interval moment method, the interval bounds of the probabilistic characteristics of the uncertain temperature field are calculated effectively. Their membership functions are eventually reconstructed from the fuzzy decomposition theorem. By comparing the results with traditional Monte Carlo simulation, the numerical example demonstrates the feasibility and effectiveness of the proposed method for solving hybrid uncertain heat conduction problems in engineering.

1. Introduction

Thermal analysis has undergone a rapid development in engineering, especially in the field of aeronautics and astronautics, where the coupled interaction of structure and heat conduction is playing a more and more significant role. Because of the complexity, the finite element method (FEM), where the complicated domains are discretized into finite small elements, has become a powerful tool for solving the heat transfer problem in the last several decades [1]. Traditional thermal analysis has been conducted under the assumption that the physical properties and boundary conditions are deterministic. However, the uncertainties in input parameters which arise from the manufacturing tolerances, measurement errors and aggressive environmental factors are unavoidable, which will lead to the uncertain temperature field [2,3].

Probabilistic methods have been considered as the most valuable approaches to model the uncertainties using random parameters whose probability functions are defined unambiguously. Various probabilistic methods have been developed for the propagation of variability through the heat transfer model. Monte Carlo simulation is the simplest approach, and is almost suitable for any stochastic heat conduction problem [4]. However, its accuracy depends on the large number of process samples. Because of the excessive computational cost, Monte Carlo method is commonly introduced as a referenced approach for validating the accuracy of other numerical methods, not as an engineering method. The stochastic perturbation method, which utilizes the Taylor expansion, is considered as an alternative approach for the heat conduction problem with random parameters [5]. Based on the generalized polynomial chaos, Xiu and Karniadakis [6] presented a spectral stochastic method for the solution of transient heat conduction subjected to random inputs. Recently, the finite difference method is introduced into the uncertain temperature field prediction with random physical parameters and initial/boundary conditions [7]. However, the probabilistic methods require the precise probability distribution functions of the uncertain parameters, which depend heavily upon a great amount of statistical information or experiment data. Unfortunately, for practical engineering problems, it is often too difficult or costly to collect sufficient information about the uncertainty.

The fuzzy set theory, introduced by Zadeh [8], provides another efficient category to represent the uncertain parameter using membership function constructed from the available expert

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opinions [9,10]. In the context of computational mathematics, the fuzzy approaches aim to obtain the membership functions of the output quantities given the membership functions of input uncertain variables. Based on the level-cut strategy, the membership function range is subdivided into a number of cut levels, and then the original fuzzy parameters can be converted into interval variables. Therefore, the fuzzy uncertainty propagation involves the application of interval analysis at the selected cut levels. Up to now, there are two main kinds of approaches for the fuzzy analysis. The first one is known as the optimization-based approach, where two global optimization problems aiming at the maximum and minimum values of the response function will be solved for each cut level [11,12]. Although the computational accuracy is high, the huge computational efforts caused by the large number of optimization problems under different cut levels embarrass its practical application in engineering. The second one is known as the interval-based approach, in which the classical interval arithmetic is used to solve the interval uncertain problem transformed from the level-cut operation [13–15]. In the interval-based fuzzy methods, the computational cost of fuzzy perturbation approach is the smallest, and the convergence condition which is related to the ranges of λ-cut intervals can be easily guaranteed. Nevertheless, the unpredictable effect caused by neglecting the high order terms is the inherent disadvantage of traditional perturbation theory [16]. But its computational accuracy for the problem with small uncertainty level is completely acceptable [17].

Theoretically, the stochastic methods and the fuzzy approaches have their own merits and deficiencies. The former is feasible for the cases with defined probability distributions, while the latter is more suitable to the situations with membership functions. Numerous researches on uncertain problems have been carried out by using single type of uncertainty modeling [18,19]. However, in practical engineering problems, both the random and fuzzy parameters may exist simultaneously. Thus, it is desirable to develop a hybrid framework which integrates the merits of both methods. Jing et al. [20] presented a novel hybrid fuzzy stochastic analytical hierarchy process approach to aid decision making by incorporating fuzzy and stochastic uncertainty into the traditional analytic hierarchy process. Sniady et al. [21] investigated a complex dynamic problem, which concerns a structure with fuzzy and random parameters in the load process. Ni and Qiu [22] proposed a new hybrid reliability model which contains randomness, fuzziness and non-probabilistic uncertainty based on the structural fuzzy random reliability and non-probabilistic set-based models. Compared with the existing pure random model and pure fuzzy model, the hybrid framework shows broader applicability in engineering. Although there has been a growing interest in mixed uncertainty analysis [23,24], it should be noted that current research on hybrid uncertain problems with random and fuzzy parameters is still in the preliminary stage and mainly concentrated in the structural analysis, while the application in the heat transfer is yet unexplored. Meanwhile, considering the advantages of perturbation theory and moment method in fuzzy theory and probabilistic analysis, it is promising to develop a new numerical method for the hybrid uncertainty propagation by using above two techniques.

The outline of the paper is as follows. The finite element equilibrium equation for the heat conduction problem is firstly established in Section 2. By using the level-cut strategy to transform the fuzzy parameters into interval variables, Section 3 presents a perturbation method to express the interval random temperature field. In Section 4, the random interval moment method, monotonicity analysis and fuzzy decomposition theorem are applied to calculate the membership functions of the probabilistic characteristics of the uncertain temperature field. Section 5 provides a numerical example to demonstrate the effectiveness of the proposed method, and lastly conclusions are drawn.

2. FEM equilibrium equation

For a three-dimensional steady-state heat conduction problem with a heat source, the governing equation can be expressed as

$$k\nabla^2 T = Q(x, y, z) \tag{1}$$

where ∇^2 denotes the Laplace operator; $T = T(x, y, z)$ is the temperature field; k stands for the heat conductivity, and $Q(x, y, z)$ is the intensity of the heat source.

For the interior domain Ω bounded by Γ as shown in Fig. 1, four kinds of boundary conditions are considered as follows

$$T|_{\Gamma_1} = T_s \tag{2}$$

$$-k\frac{\partial T}{\partial \mathbf{n}}\bigg|_{\Gamma_2} = q_s \tag{3}$$

$$-k\frac{\partial T}{\partial \mathbf{n}}\bigg|_{\Gamma_3} = h(T - T_e) \tag{4}$$

$$-k\frac{\partial T}{\partial \mathbf{n}}\bigg|_{\Gamma_4} = \sigma\varepsilon(T^4 - T_e^4) \tag{5}$$

where T_s is the given boundary temperature; $\mathbf{n}$ is the normal vector of the boundary; q_s stands for the boundary heat flux; h denotes the heat transfer coefficient; T_e is the ambient temperature; σ represents the Stefan–Boltzmann constant, and ε stands for the surface emissivity.

By discretizing the interior domain Ω into M isoparametric elements, the equilibrium equation for the heat conduction analysis in finite element framework can be expressed in the following general form

$$\mathbf{KT} = \mathbf{R} \tag{6}$$

where $\mathbf{K}$ and $\mathbf{R}$ stand for the global heat conductivity matrix and equivalent nodal heat flow vector, respectively. They can be obtained by assembling all the element heat conductivity matrix $\mathbf{K}_e$ and element heat flow vector $\mathbf{R}_e$, which are derived in the discrete elements

$$\mathbf{K} = \sum_{e=1}^{M}\mathbf{K}_e, \quad \mathbf{R} = \sum_{e=1}^{M}\mathbf{R}_e \tag{7}$$

$$\begin{aligned}\mathbf{K}_e &= \int_{\Omega_e} k(\nabla\mathbf{N})\cdot(\nabla\mathbf{N})^T d\Omega + \int_{\Gamma_3\cap\Omega_e} h\mathbf{N}\cdot\mathbf{N}^T d\Gamma \\ \mathbf{R}_e &= \int_{\Omega_e} Q\mathbf{N}d\Omega - \int_{\Gamma_2\cap\Omega_e} q_s\mathbf{N}d\Gamma + \int_{\Gamma_3\cap\Omega_e} hT_e\mathbf{N}d\Gamma - \int_{\Gamma_4\cap\Omega_e} \sigma\varepsilon(T^4 - T_e^4)\mathbf{N}d\Gamma\end{aligned} \tag{8}$$

where $\mathbf{N}$ is the row vector of the Lagrange interpolation shape function.

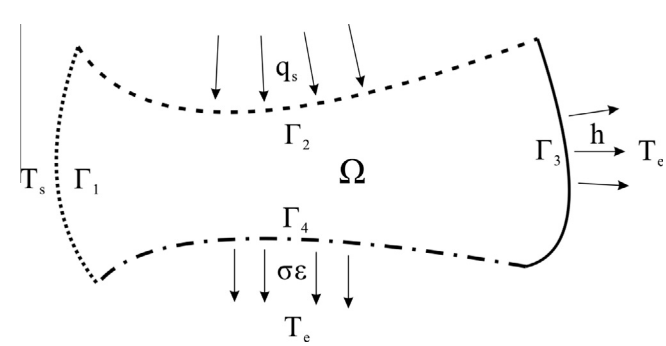

Fig. 1. Four kinds of boundary conditions.