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The robustness of the permeability of constructal tree-shaped fissures

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ABSTRACT

Here we develop analytically the formulas for effective permeability in several configurations using the closed-form description of tree networks designed to provide flow access. The objective was to find the relation between the permeability and porosity of tree-shaped fissures. We found the effect of the fracture size on the permeability for fixed number of bifurcation and the results showed that the permeability of the fracture network increased rapidly with the size of the fracture. Next, we found a relation between the Reservoir Quality Index (RQI) and the porosity of the fracture. The results in this paper have been validated by comparison with experimental and numerical results. We show that the permeability formulas do not vary much from one tree design to the next, suggesting that similar formulas may apply to naturally fissured porous media with unknown precise details, which occur in natural reservoirs.

1. Multi-scale flow structures distributed non uniformly

Fractured porous media such as hot dry-rock deposits [1–3] are considerably more complicated and difficult to describe than the packed-spheres systems described by compact formulas such as the Carman–Kozeny permeability model [4]. The reason is that fractured media have fissures with many length scales, which are distributed and oriented nonuniformly through the available volume. The fluid that is forced to permeate through such a medium follows paths that can be described as random combinations of organized channel flow and disorganized porous medium flow. The organized portion is the flow through the largest fissures, while the disorganized flow is the seepage through the smaller volumes filled with the finer fissures.

The challenge is to describe the permeability of such complex flow structures by means that approach the simplicity of the models known for uniform packings of particles with a single length scale. Here we show that the flow permeability of fractured solid has relatively universal and predictable characteristics. We show this by modeling the multi-scale flow volume as tree shaped, with a precise architecture that results from constructal design: the maximization of flow access through the fluid space [5,6].

A new trend in convection and designed porous media is the use of tree-shaped structures for accessing areas or volumes with minimal flow resistance. The current progress was reviewed in Ref. [7]. One development that is relevant to the present paper is the design of nearly-optimal dendritic structures based on compact recurrence formulas obtained by minimizing the length of every link in the dendritic flow structure [8]. Another relevant development is the demonstrated robustness [9] of dendritic flow configurations: the performance level does not vary significantly from one configuration to another.

The current progress made on dendritic structures is an opportunity to describe in compact form the 'volumetric permeability' of such structures. This may shed light on the constitution of the effective permeability of a naturally fractured porous medium. The basis for this speculation is that dendritic flow structures are similar to fissured porous media. They have channels with multiple length scales, which are organized hierarchically. These channels are distributed and oriented nonuniformly. The robustness of dendritic structures suggests that the performance of similarly complex structures (e.g., fissured media) is similar to that of dendritic structures, and may be expressed by formulas developed for dendritic structures. Progress in this direction was demonstrated by Xu et al. [10] who calculated the permeability of a fractal-like tree network in one configuration, fractal-like tree networks between one point and a straight line. A fractal-like tree is a geometric object with postulated (assumed, not predicted) algorithm of construction and smallest length scale. Several other investigators have studied self-similar fracture networks. For example, Velde et al. and Sahimi et al. [11–14] analyzed the fractal patterns of fractures in porous media. Gueguen and Chelidze [15] also studied the fracture patterns in Stockbridge dolomite marble,

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Nomenclature

D	channel thickness, m
i	bifurcation level
K	permeability, m^2
L	channel length, m
L_d	distance from the center to the side of the configuration, m
$\dot{m}$	mass flow rate, kg/s
P	pressure, Pa
RQI	Reservoir Quality Index
u	volume average velocity, m/s
V	volume, m^3
W	channel width, m

Greek symbols

θ	angle, degree
ϕ	porosity
ρ	density, kg/m^3
ν	kinematic viscosity, m^2/s
μ	dynamic viscosity, N s/m^2

Subscripts

c	center
n	largest scale
p	pore
t	total
0	smallest scale

which reported a highly interconnected and very branched network. Wang et al. [16] and Voorn et al. [17] reported numerically the permeability of fractured porous media. Miao et al. [18] studied analytically the permeability of fractured rocks.

In this paper we explore the theoretical basis for the permeability of fissures by taking full advantage of the closed-form description of tree networks with minimal-length channels [8]. We develop analytically the formulas for effective permeability in four different flow configurations: tree connecting one point and a square frame, flow perpendicular to the plane of the tree, tree connecting a circle with its center, and tree connecting a point with an area. To study the effect of changing the flow direction (along the plane of the design or normal to the plane) is important because the assembly of fissures is not necessarily the result of the flow itself. Fissures in rocks are not like the river channels on the plain. They can be of solid mechanics origin, such as the phenomenon of cracking during the release of tension in a solid that is in a state of volumetric tension. We show that the permeability formulas do not vary much from case to case, suggesting that essentially the same broad formula applies to naturally fissured porous media.

2. Tree connecting one point and a square frame

Start with is the two-dimensional tree structure shown in Fig. 1. The flow connects the center of the square with many ports positioned equidistantly on the perimeter. Each channel is modeled as the space between two parallel plates of spacing D_i and flow length L_i, where i indicates the pairing or bifurcation level. The smallest scale is found near the perimeter, $i = 0$. The structure has n levels of pairing. The largest channels reach the center, $i = n$. Symmetry allows us to focus on a one-eighth section of the structure.

The optimal configuration is discovered numerically and sketched qualitatively as Section 2in Fig. 1: noteworthy is the longest channel, which is not aligned with the diagonal of the square. The alternative shown as Section 1is the structure based on minimizing the length of each channel, starting from the smallest (L_0) and proceeding towards the largest [8]. This structure is characterized by the relations

$$L_i = 2^i L_0 \qquad (1)$$

$$n_i = 2^{n-i} \qquad (2)$$

$$\dot{m}'_i = 2^i \dot{m}'_0 \qquad (3)$$

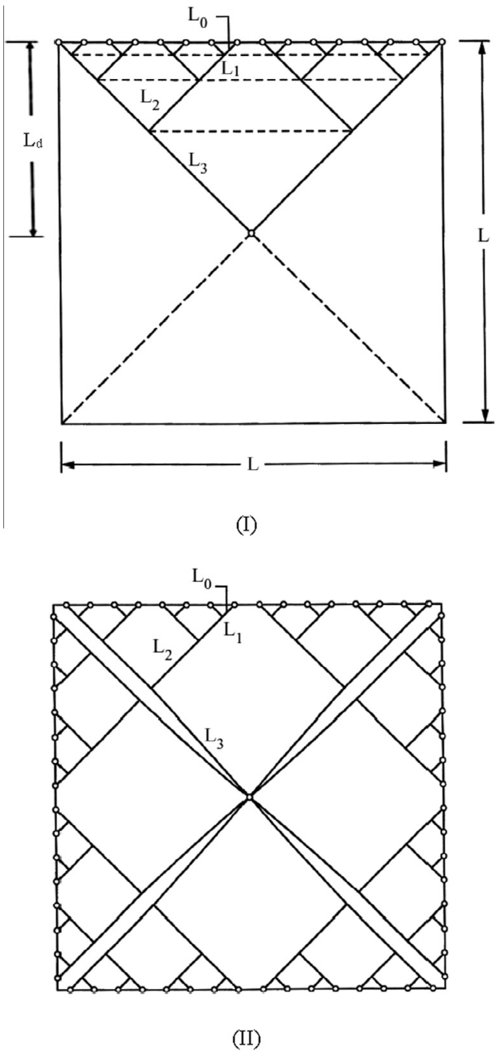

Fig. 1. Tree-shaped network connecting a square frame with its center.

where $i = 0, 1, \ldots, n$. Furthermore, n_i is the number of channels of size (D_i, L_i), and $\dot{m}'_i$ is the mass flow rate (kg s^{-1} m^{-1}) through a channel at level i. The number of ports on one eighth of the square