International Journal of Heat and Mass Transfer 93 (2016) 1100–1106

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Unsteady thermal boundary layer flows of a Bingham fluid in a porous medium following a sudden change in surface heat flux

D. Andrew S. Rees [a,*], Andrew P. Bassom [b,1]

[a] Department of Mechanical Engineering, University of Bath, Bath BA2 7AY, UK
[b] School of Mathematics and Statistics, The University of Western Australia, Crawley, WA 6009, Australia

ARTICLE INFO

Article history:
Received 30 July 2015
Received in revised form 9 October 2015
Accepted 9 October 2015
Available online 21 November 2015

Keywords:
Porous media
Boundary layer
Unsteady flow
Convection
Bingham fluid
Yield stress

ABSTRACT

We consider the effect of suddenly applying a uniform heat flux to a vertical wall bounding a porous medium which is saturated by a Bingham fluid. We consider both an infinite porous domain and a vertical channel of finite width. Initially, the evolving temperature field provides too little buoyancy force to overcome the yield threshold of the fluid. For the infinite domain convection will always eventually arise, but this does not necessarily happen in the vertical channel. We show (i) how the presence of yield surfaces alters the classical results for Newtonian flows and (ii) the manner in which the locations of the yield surfaces change as time progresses.

1. Introduction

Bingham fluids are an example of a yield stress fluid. Unlike Herschel–Bulkeley and Casson fluids, they exhibit a linear stress–strain relationship once the yield stress is exceeded. They arise in a wide variety of situations both in the environment and industry, and examples of the very many natural and man-made fluids which exhibit a yield stress have been collated and prsented in the chapter [1].

The aim of the present short paper is to investigate the manner in which convection arises when a Bingham fluid saturates a porous medium. There already exist some papers on this type of topic, but they are concerned with the equivalent fluid problem, i.e. there is no porous matrix present. We refer to the analyses by Yang and Yeh [2] and Bayazitoglu et al. [3] who studied free convection in a sidewall-heated channel. They find that steady convection will only arise whenever the Rayleigh number is sufficiently large that the buoyancy force is then able to overcome the yield stress. When flow occurs the velocity profile consists of five regions, with three regions of flow alternating with two of plug flow (i.e. constant velocity with no shear). The plug flow regions are themselves moving and are placed at equal distances either side of the centreline of the channel. Other papers which consider variations on this theme are those by Patel and Ingham [4] who consider a mixed convection with the combination of buoyancy and a driving pressure gradient, Barletta and Magyari [5] who consider a free convection variant on vertical Couette flow, and Karimfazli and Frigaard [6] whose study of free convection when the boundary temperatures vary linearly with distance up the walls. The unsteady analysis of Kleppe and Marner [7] is also important because it considers the evolution of the temperature and velocity profiles in a vertical channel after an sudden change in temperature of one of the vertical walls.

The present paper considers the unsteady unidirection convection which is set up by suddenly changing the boundary heat flux of a vertical surface. This is a natural extension to the work of Rees and Bassom [8] who studied an impulsive change in the boundary temperature. We will see that the final outcome here has many qualititative differences from those found in [8]. Again, we will consider both a semi-infinite domain (i.e. bounded by a single vertical wall), and a vertical channel of constant thickness. As in [8] buoyancy forces spread into the porous medium due to the diffusion of heat from the heated surface. We find that convection does not happen at first, but that there is an onset time after which convection persists. We present detailed exact solutions for the locations of the yield surfaces and overall velocity flux, and an asymptotic analysis yields highly accurate data for large times.

* Corresponding author.
E-mail addresses: D.A.S.Rees@bath.ac.uk (D. Andrew S. Rees), andrew.bassom@uwa.edu.au (A.P. Bassom).
[1] Permanent address: School of Mathematics and Physics, University of Tasmania, Private Bag 37, Hobart, TAS 7001, Australia.

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.10.021

Nomenclature

Latin letters

A_n	coefficients
B_n	coefficients
$\mathcal{D}$	equal to $\ln\delta$
$f(\eta)$	similarity solution
g	gravity
G	threshold body force
K	permeability
L	length scale
n	summation index
p	pressure
p_x	pressure gradient in the x-direction
q	imposed surface heat flux
Q	total vertical velocity flux
Ra	Darcy–Rayleigh number
Rb	Rees–Bingham number
t	time
T	temperature (dimensional)
T_0	ambient (cold) temperature
T_1	temperature of heated surface
u	vertical Darcy velocity
x	vertical coordinate
y	horizontal coordinate

Greek letters

α	thermal diffusivity
β	coefficient of cubical expansion
δ	transformed time (Eq. (39))
η	similarity variable
η_1	location of left hand yield surface
η_2	location of right hand yield surface
η_3	value of η which is equivalent to $y = 1$
η_y	location of yield surface
θ	temperature (nondimensional)
μ	dynamic viscosity
ρ	reference density
σ	heat capacity ratio

Other symbols

–	dimensional quantities
max	maximum value
min	minimum value
y	yield

2. Governing equations

We follow the early paper by Pascal [9] which employs a threshold gradient model to describe the one-dimensional flow of a Bingham fluid in a porous medium:

$$\bar{u} = \begin{cases} -\frac{K}{\mu}\left[1 - \frac{G}{|\bar{p}_{\bar{x}}|}\right]\bar{p}_{\bar{x}} & \text{when} \quad |\bar{p}_{\bar{x}}| > G, \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where G denotes the threshold gradient (or, more generally, the threshold body force) above which the fluid yields. When buoyancy is included as an extra body force, the threshold model becomes,

$$\bar{u} = \begin{cases} -\frac{K}{\mu}\left[1 - \frac{G}{|\bar{p}_{\bar{x}} - \rho g\beta(T-T_0)|}\right](\bar{p}_{\bar{x}} - \rho g\beta(T-T_0)) & \text{when} \quad |\bar{p}_{\bar{x}} - \rho g\beta(T-T_0)| > G, \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

and where $\bar{x}$ is now the vertical coordinate and $\bar{u}$ is the corresponding Darcy velocity. We have assumed that the Boussinesq approximation applies when writing down the buoyancy term, and T_0 is the initial temperature of the porous medium. If a heated vertical surface is of infinite extent in both the positive and negative $\bar{x}$-directions, then there will be a zero horizontal fluid velocity. We may allow both $\bar{u}$ and T to be functions only of the horizontal coordinate, $\bar{y}$, and time, $\bar{t}$, and so the equation of continuity is satisfied and the heat transport equation is,

$$\sigma T_{\bar{t}} = \alpha T_{\bar{y}\bar{y}}, \tag{3}$$

where σ is heat capacity ratio between the porous medium and the saturating fluid, and α is the effective thermal diffusivity of the saturated porous medium. At $\bar{t} = 0$ a uniform and steady heat flux, q, is applied suddenly to the vertical bounding surface:

$$k\frac{\partial T}{\partial \bar{y}} = -q, \tag{4}$$

where k is the thermal conductivity of the medium.

Equations (2) and (3) may be nondimensionalised using the scalings,

$$(\bar{x},\bar{y}) = L(x,y), \quad \bar{u} = \frac{\alpha}{L}u, \quad \bar{p} = \frac{\alpha\mu}{K}p, \quad T = T_0 + \frac{qL}{k}\theta,$$

$$\bar{t} = \frac{\sigma L^2}{\alpha}t, \quad G = \frac{\alpha\mu}{KL}\mathrm{Rb}, \tag{5}$$

and we obtain,

$$u = \begin{cases} \mathrm{Ra}\,\theta - p_x - \mathrm{Rb}, & \mathrm{Rb} < \mathrm{Ra}\,\theta - p_x, \\ 0, & -\mathrm{Rb} < \mathrm{Ra}\,\theta - p_x < \mathrm{Rb}, \\ \mathrm{Ra}\,\theta - p_x + \mathrm{Rb}, & \mathrm{Ra}\,\theta - p_x < -\mathrm{Rb}, \end{cases} \tag{6}$$

and

$$\theta_t = \theta_{yy}. \tag{7}$$

In the above the Darcy–Rayleigh number is given by

$$\mathrm{Ra} = \frac{\rho g\beta q K L^2}{k\mu\alpha}, \tag{8}$$

and the parameter, Rb, is given by

$$\mathrm{Rb} = \frac{KL}{\mu\alpha}G. \tag{9}$$

This latter parameter is a scaled version of the yield pressure gradient, G, and might be referred to as a porous convective Bingham number; hereinafter it is termed the Rees–Bingham number [1].

The lengthscale, L, which was introduced in Eq. (5), will be taken to be the dimensional width of the vertical channel when that configuration is being studied. But when the porous medium occupies a semi-infinite domain there is no natural external lengthscale that may be used. Therefore we set the Darcy–Rayleigh number to a unit value, which will then automatically define a natural lengthscale, L, in terms of the properties of the medium; thus we have,

$$L = \sqrt{\frac{\mu\alpha}{\rho g\beta q K}}. \tag{10}$$