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Soret effect on the double diffusive convection instability due to viscous dissipation in a horizontal porous channel

Kamalika Roy *, P.V.S.N. Murthy

Department of Mathematics, Indian Institute of Technology Kharagpur, WB 721302, India

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ABSTRACT

The effect of the Soret parameter on the convective stability of double diffusive convection solely because of the viscous dissipation in a horizontal porous channel is studied. The lower boundary is adiabatic whereas the upper boundary is considered to be isothermal. The convective stability of the present system is governed by the solutal Rayleigh number (R) and is influenced by the viscous dissipation parameter (ξ), Lewis number (Le) and Soret parameter (Sr). For non positive values of the Soret parameter, the longitudinal rolls happen to be the most unstable ones when ξ takes small values. With positive values of the Soret parameter, the transverse rolls are seen to be the most unstable for relatively smaller values of the viscous dissipation parameter. As the viscous dissipation effect becomes stronger, the longitudinal rolls become the most unstable ones even for positive Soret parameter and the transverse rolls become more unstable for non-positive Soret parameter. It is observed that the Soret parameter has significant effect on convective instability and this is discussed. It has also been noticed that viscous dissipation shows a dual effect in presence of the Soret effect. For fixed values of the viscous dissipation parameter and Lewis number, negative values for the Soret number advances the onset of convection. Though positive values of the Soret parameter stabilizes the flow with smaller values of the Lewis number, but it destabilizes the flow when Lewis number and viscous dissipation parameter take larger values.

1. Introduction

A vast number of research articles dealing with the natural convection in a horizontal porous layer, induced either by horizontal or vertical temperature gradients can be found in the literature. Nield et al. [1] studied the convection induced by inclined thermal and solutal gradients in a shallow horizontal layer of a porous medium. Mojtabi et al. [2], Nield and Bejan [3] give a comprehensive idea about double-diffusive convection in porous media.

It is seen from the literature that in binary fluids, the cross coupling between thermal and solutal gradients is significant especially when the thermal gradients are more prominent. Owing to the importance of the Soret effect in many physical processes its effect on the double diffusive convection in porous media has attracted the attention of several researcher in the recent past. Hurle and Jakeman [4] discussed the Soret effect on the thermosolutal Rayleigh–Jeffrey's problem. Considering a quiescent layer of fluid bounded by rigid boundaries, Patil and Rudraiah [5] discussed the Soret effect on double diffusive convection using linear theory. Brand and Steinberg [6] discussed the double diffusive convection in a porous layer saturated with binary fluid and heated from below or above. They first mentioned that in presence of the Soret parameter, oscillatory convection can set in even if the fluid mixture is heated from above. Later in 2003, Bahloul et al. [7] analyzed convection induced by the Soret effect in a shallow horizontal porous layer. Narayana et al. [8] analyzed the Soret effect in a horizontal porous layer with inclined thermal and solutal gradients. Linear and non-linear stability analysis of double diffusive convection in a fluid saturated anisotropic porous layer with Soret effect has been carried out by Gaikwad et al. [9]. It brings out the significance of the mechanical and thermal anisotropy of the medium in presence of the Soret coupling in the system. Mojtabi et al. [10] has studied the stability of the convection induced by Soret effect in a porous medium while Wang and Tan [11] discussed the double diffusive convection of Maxwell fluid in porous media considering the Soret effect. Altawallbeh et al. [12] have done both linear and non-linear stability analysis to show the effect of the Soret parameter in the double diffusive convection in an anisotropic porous layer with internal heat source and heated from below. The Soret effect on the convective stability in a porous layer saturated by visco-elastic fluid has been studied by Gaikwad and Dhanraj [13] by means of both linear and non-linear stability analysis. The linear stability analysis of

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* Corresponding author.
E-mail address: komolika.roy89@gmail.com (K. Roy).

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ferrothermohaline convection in an anisotropic porous layer with significant Soret effect has been done by Sekar et al. [14]. Considering an anisotropic and sparsely packed porous medium, Gaikwad and Kamble [15] has studied the effect of the Soret parameter on the convective instability of the medium. It has been noticed in all these studies that the Soret parameter has significant effect on the convective stability.

It is evident from the literature, that in mixed convection or in vigorous natural convection processes, viscous dissipation is more significant. In 1962, the effect of dissipation parameter on natural convection with isothermal boundary and boundary with constant heat flux is studied by Gebhart [16]. This dissipation number is independent of Prantdl number and Grashof number. Later in 1969, the effects of viscous dissipation on external natural convection is analyzed by Gebhart and Mollendorf [17]. Turcotte et al. [18] investigated the effect of viscous dissipation parameter on finite amplitude Bénard convection. Off late, it has been argued that the viscous dissipation could be the sole mechanism for the convective flow in double diffusive processes. From the very recent review article by Barletta [19] and also from some more earlier works by his group of researchers (some of which are cited below), who have modeled the double diffusive convection induced by viscous dissipation, established the destabilizing effect of the convective flow due to viscous dissipation in the medium. Barletta et al. [20] presented the linear stability analysis of the thermal convection induced solely by viscous dissipation in a horizontal porous layer. Barletta et al. [21] considered a porous medium with horizontal temperature gradient along the upper boundary and analyzed the effect of viscous dissipation on thermal convection in the medium. The effect of viscous dissipation on the linear stability of double diffusive convection has been addressed by Barletta and Nield [22] while the significance of viscosity variations on the dissipative instability in a porous layer with horizontal throughflow has been presented in Barletta and Nield [23]. Recently Barletta et al. [24] considered permeable boundaries instead of impermeable boundaries and has done the linear stability analysis. Barletta and Storesletten [25] studied the linear stability analysis of forced convection in a vertical porous cylinder. When there is external heat transfer coefficient, Biot number has a significant role in determining the convection. Storesletten and Barletta [26] also studied mixed convection in a porous medium induced by viscous-dissipation. Effect of the viscous dissipation on the stability of mixed convection in a porous layer saturated by viscoelastic (Oldroyd-B model) fluid is discussed by Alves et al. [27]. It is seen that viscous dissipation has a destabilizing effect.

It will be interesting to investigate the effect of the Soret parameter on the problem considered in Barletta and Nield [22]. In most of the cases viscous dissipation showed a de-stabilizing effect when the Soret effect is neglected. It is still unknown how the viscous-dissipation reacts in presence of the Soret parameter. Also in the case of cross diffusion, Lewis number plays an important role. This Soret coupling between the thermal and solutal fields not only brings in mathematical complexity to the linear stability analysis, but also reveals several interesting features of double diffusive convection instability due to viscous dissipation in a horizontal porous medium. The Soret effect brings in an interplay between thermal buoyancy and solutal buoyancy. Thus the convective instability is acted upon by the interplay between the various governing parameters such as ξ that is associated with viscous dissipation, solutal Rayleigh number (R), Lewis number (Le) and Soret parameter (Sr).

Investigating the effect of Soret parameter on the similar configuration with non zero concentration flux at the bottom wall and isoconcentration at the top and bottom wall, which is further more deviation from what has been done in Barletta and Nield [22], will be another challenging problem, but this will complicate the mathematical treatment of linear stability of double diffusive convection due to viscous dissipation.

2. Mathematical formulation

A binary fluid saturated porous layer of thickness L which is bounded by two horizontal walls at $\bar{y} = 0$ and $\bar{y} = L$ is considered as shown in Fig. 1.

The lower wall is adiabatic and the upper one is considered to be isothermal. Both these boundaries are impermeable. The boundaries have uniform concentration and is denoted by $\overline{C}$. Here 'bar' denotes dimensional quantities. The gravitational acceleration is $\bar{\mathbf{g}} = -g\mathbf{e}_y$.

The fluid motion is governed by the Darcy law. Viscous dissipation and Soret effect are assumed to be prominent in the medium. As there is no heating from outside of the medium, it is the internal heating associated with the viscous dissipation that directs the convection in the medium. Also, a concurrent mass diffusion that is present due to the concentration difference maintained at the horizontal walls gets effected due to the thermal gradients contributing to the concentration distribution (due to the Soret effect) in the medium. Also, the Oberbeck–Boussinesq approximation is applicable. Under these assumptions the governing equations for the heat and solutal transport is given by

$$\bar{\nabla} \cdot \bar{\mathbf{u}} = 0 \tag{1}$$

$$\frac{\mu}{K}\bar{\mathbf{u}} = -\bar{\nabla}\bar{p} + \rho_L g[\beta_T(\overline{T} - T_L) + \beta_C(\overline{C} - C_L)]\mathbf{e}_y \tag{2}$$

$$\sigma\frac{\partial \overline{T}}{\partial \bar{t}} + \bar{\mathbf{u}} \cdot \bar{\nabla}\overline{T} = \alpha\bar{\nabla}^2\overline{T} + \frac{\nu}{Kc}\bar{\mathbf{u}} \cdot \bar{\mathbf{u}} \tag{3}$$

$$\phi\frac{\partial \overline{C}}{\partial \bar{t}} + \bar{\mathbf{u}} \cdot \bar{\nabla}\overline{C} = D\bar{\nabla}^2\overline{C} + D_{CT}\bar{\nabla}^2\overline{T} \tag{4}$$

and the boundary conditions are

$$\bar{y} = 0: \quad \bar{v} = 0, \quad \frac{\partial \overline{T}}{\partial \bar{y}} = 0, \quad \overline{C} = C_B \tag{5}$$

$$\bar{y} = L: \quad \bar{v} = 0, \quad \overline{T} = \bar{T}_L, \quad \overline{C} = C_L \tag{6}$$

where $\bar{\mathbf{u}} = (\bar{u}, \bar{v}, \bar{w})$ is the velocity, ϕ is the porosity of the layer, σ is the heat capacity ratio, ρ_L is reference density, c is the heat capacity per unit mass of the fluid, β_T and β_C are thermal and concentration

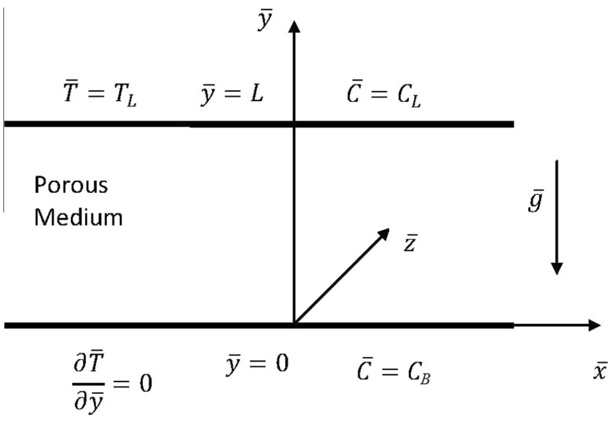

Fig. 1. Physical configuration of the system.