International Journal of Heat and Mass Transfer 90 (2015) 364–375

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

An inverse problem in determining the optimum shapes for partially wet annular fins based on efficiency maximization

Cheng-Hung Huang *, Yun-Lung Chung

Department of Systems and Naval Mechatronic Engineering, National Cheng Kung University, Tainan 701, Taiwan, ROC

ARTICLE INFO

Article history:
Received 25 December 2014
Received in revised form 22 June 2015
Accepted 23 June 2015

Keywords:
Inverse problem
Partially wet annular fins
Fin shape optimization
Conjugate gradient method

ABSTRACT

The optimum shapes of partially wet annular fins adhered to a bare tube are determined in this work based on the desired fin efficiency and fin volume with the conjugate gradient method (CGM). The major advantage of utilizing the CGM in the present inverse fin design problem lies in that no *a priori* information on the functional form of the fin shape is needed; the optimal fin shape can still be obtained automatically during the iterative process. It is assumed that the surrounding air has relative humidity between 70% and 90%, which will result in a partially wet annular fin. The validity of the present inverse design problem using CGM is examined based on numerical experiments. The numerical results show that as the fin volume V increases, the relative humidity ϕ increases, the conductivity ratio G increases, Bi_i increases, and Bi_o decreases, both the interface radius between the wet fin region and the dry fin region r_{wd} and the fin efficiency η will also increase. Finally, it is concluded that the estimated optimum annular fin shape always has the highest fin efficiency among all five common annular fins, and this conclusion implies that more heat can be carried away into the environment using the present optimum fins.

1. Introduction

Annular finned-tube heat exchangers are widely used in applications of air-conditioning and refrigeration systems. For instance, the fin surface temperature is generally below the dew point temperature in evaporators, and as a result, dehumidification of the air occurs. With dehumidification, the fin surface is wetted, and heat and mass transfer occur simultaneously. The fin surface can be classified as dry, partially wet, or fully wet depending on the base and boundary temperatures of the fin and the dew point of the surrounding air. Many studies have been published on the heat and mass transfer characteristics of fin-and-tube heat exchangers under dehumidifying conditions, and the focus of this work is placed on designing the optimum shape of partially wet annular fins.

Other studies have considered factors such as the temperature distribution on the fin surface and the fin efficiency. For example, Naphon [1] examined the heat transfer characteristics of an annular fin under dry, partially wet, and fully wet surface conditions and presented theoretical results of the heat transfer characteristics and the fin efficiency of the annular fin. The results obtained from the model were validated by comparing them with those obtained by other researchers. Wu and Bong [2] were the first to derive an analytical solution for the efficiency of a longitudinal straight fin under dry, fully wet, and partially wet surface conditions. They indicated that only when the fin is partially wet does the overall fin efficiency strongly depend on the relative humidity.

Rosario and Rahman [3] presented an analysis of heat transfer in a partially wet annular fin assembly during dehumidification. In their study, the enhancement of heat transfer due to condensation was found to be quite significant. Pirompugd et al. [4] utilized the "finite circular fin method" (FCFM) to analyze the performance of fin-and-tube heat exchangers having plain fin configuration under dehumidifying conditions. Correlations applicable for both fully wet and partially wet conditions are proposed to describe the heat and mass performance for their plain fin configuration.

The primary concern in fin design is the fin volume (or weight) and fin efficiency, and it is highly desirable to optimize the shape of the fins based on these two constraints. The optimum fin dimensions are those for which the maximum (or desired) fin efficiency is obtained under the constraint of specified fin volume.

Numerous fin-design problems have been examined to optimize the dimensions of wet annular fins for various profiles. Brown [5] examined the optimum dimensions of uniform fully wet annular fins. Finally, a comparison was made between the optimum dimensions of the uniform fins and the dimensions of fins with minimal weight. Kang [6] utilized a variation separation

* Corresponding author.
E-mail address: chhuang@mail.ncku.edu.tw (C.-H. Huang).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.06.070
0017-9310/

Nomenclature

$\bar{A}(\bar{r})$	cross-sectional area of the fully wet annular fin [m^2]
Bi_a	Biot number of ambient air
Bi_i	Biot number on the inner bare tube
Bi_o	Biot number on the external bare tube
$\bar{h}_d$	mass transfer coefficient [kg m^{-2} s^{-1}]
$\bar{h}_{fg}$	latent heat of condensation of moisture [J kg^{-1}]
$\bar{C}_{p,a}$	constant pressure specific heat for moist air [J kg^{-1} K^{-1}]
G	thermal conductivity ratio
ϕ	relative humidity
$\bar{h}$	convective heat transfer coefficient [W m^{-2} K^{-1}]
J	functional defined by Eq. (3)
J'	gradient of functional defined by Eq. (15)
$\bar{k}$	thermal conductivity [W m^{-1} K^{-1}]
L_e	Lewis number
$\bar{P}$	half fin pitch [m]
q	actual heat transfer rate through the partially wet annular fin
Q	ideal heat transfer rate through the partially wet annular fin
$\bar{r}_b$	inner radius of the bare tube [m]
$\bar{r}_o$	external radius of the bare tube [m]
$\bar{r}_{wd}$	interface radius between wet and dry fin regions [m]
$\bar{r}_t$	external radius of the partially wet annular fin [m]
$\bar{S}(\bar{r})$	perimeter of the partially wet annular fin [m]
$\bar{T}_w$	temperature of the bare tube [°C]
$\bar{T}_{f,w}$	temperature of the wet surface of the partially wet annular fin [°C]
$\bar{T}_{f,d}$	temperature of the dry surface of the partially wet annular fin [°C]
$\upsilon(\delta)$	estimated partially wet annular-fin volume
V	specified partially wet annular-fin volume

Greeks

α	weighting coefficient
β	search step size
γ	conjugate coefficient
$\delta(r)$	thickness of the partially wet annular fin
$\theta(r)$	estimated dimensionless annular fin temperature
$\Delta\theta(r)$	sensitivity function defined by Eq. (7)
$\lambda(r)$	Lagrange multiplier defined by Eq. (13)
$\Gamma(\cdot)$	Dirac delta function
η	partially wet annular fin efficiency
Φ	desired fin efficiency
ϕ	relative humidity
ε	convergence criterion
$\bar{\omega}_f$	specific humidity of air on the annular fin
$\bar{\omega}_h$	specific humidity of the surrounding air

Superscript

n	iteration index
_	dimensional quantities

method to determine the fin length of a rectangular profile fully wet annular fin to yield the optimum performance. In that study, the author concluded that, when considering the thermal conductivity to be constant, the optimum length and effectiveness are independent of the properties of the material used, whereas the optimum base thickness and the volume of the fin are inversely proportional to the thermal conductivity of the fin material.

Ullmann and Kalman [7] examined four different cross-section shapes of fully wet annular fins to find the efficiency and the optimized dimensions of fins, which were determined for a constant thickness, a constant area for heat flow, and for triangular and parabolic fin shapes. Finally, the optimized dimensions and the fin efficiency were presented in the study, thus enabling the best fin to be designed for any practical use.

In all of the above references, the optimum fin shape was determined based on either the minimum weight or the maximum heat transfer rate through the fin base. Fin-design problems based on the desired fin efficiency for a specified fin volume are very limited in the literature.

For this reason, Huang and Chung [8] developed an inverse design algorithm using the conjugate gradient method (CGM) to estimate the optimum shapes for dry annular fins, based on the desired fin efficiency and fin volume. The results indicated that the optimum fin has higher fin efficiency than those for the other common five annular fins. Moreover, it is noticed that when the Biot numbers and fin volume are varied, the optimum annular fin efficiency and shape are also subject to change. Recently, Huang and Chung [9] extended their research to determine the optimum shapes for fully wet annular fins. It is assumed that the surrounding air has an assumed relative humidity of 100%, which will result in a fully wet annular fin. The functional form of the fin shape does not need to be known before the estimation; the optimal fully wet fin shape can be obtained automatically during the iterative process.

The objective of this study aims at developing an inverse design algorithm using the conjugate gradient method (CGM) to determine the optimum shape for partially wet annular fins adhered to a bare tube, based on the desired fin efficiency and the specified fin volume. The iterative algorithm using CGM is derived based on the perturbation principle, and the solutions of resultant sensitivity and adjoint problems can be used to calculate the search step size and gradient of the functional. These three problems will be discussed and derived in the following sections.

2. The direct problem

The following partially wet annular-fin direct problem with fins adhered to a bare tube of the evaporator is considered in the present study. The relative humidity ϕ is chosen such that the dew point occurs within the annular fin and the partially wet condition annular fin results.

The dimensional energy equations as well as the boundary conditions for the steady-state partially wet annular fin with fins adhered to a bare tube of the evaporator can be obtained as follows [1]:

$$\frac{d}{d\bar{r}}\left[\bar{r}\frac{d\bar{T}_w(\bar{r})}{d\bar{r}}\right]=0;\quad \text{in } \bar{r}_b\leqslant\bar{r}\leqslant\bar{r}_o \tag{1a}$$

$$\frac{d}{d\bar{r}}\left[\bar{A}(\bar{r})\frac{d\bar{T}_{f,w}(\bar{r})}{d\bar{r}}\right]-\frac{\bar{S}(\bar{r})}{\bar{k}_f}\left\{\bar{h}_o(\bar{T}_{f,w}(\bar{r})-\bar{T}_h)+\bar{h}_{fg}\bar{h}_d\left[\bar{\omega}_f(\bar{T}_{f,w})-\bar{\omega}_h\right]\right\}=0;\quad \text{in } \bar{r}_o<\bar{r}\leqslant\bar{r}_{wd} \tag{1b}$$

$$\frac{d}{d\bar{r}}\left[\bar{A}(\bar{r})\frac{d\bar{T}_{f,d}(\bar{r})}{d\bar{r}}\right]-\frac{\bar{S}(\bar{r})}{\bar{k}_f}\left[\bar{h}_o(\bar{T}_{f,d}(\bar{r})-\bar{T}_h)\right]=0;\quad \text{in } \bar{r}_{wd}<\bar{r}\leqslant\bar{r}_t \tag{1c}$$