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Online heat flux estimation using artificial neural network as a digital filter approach

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ABSTRACT

Surface heat flux estimation using temperature measurement data from the interior points is known as inverse heat conduction problem (IHCP). Several methods have been developed as solution techniques for IHCP's including analytical and numerical approaches. Digital filter representation for IHCP solution (Woodbury and Beck, 2013; Beck et al., 1985) is one of the methods which can be used for near real-time heat flux estimation. In this study, artificial neural network (ANN) is utilized as a digital filter, for near real-time heat flux estimation using temperature measurement data. Considering temperatures as the inputs and heat flux as the output, the weights can be interpreted as filter coefficients. The proposed approach is used for both constant and temperature dependent material properties. The method developed is tested through several test cases using exact solutions and numerical models. The results show that ANN can be used as a digital filter method for near real-time surface heat flux estimation. The advantages and disadvantages of the method are also discussed.

1. Introduction

Real-time heat flux measurement has great significance in numerous industrial applications such as metal heat treating, quenching, fire safety tests, furnace operation, and more. Heat flux measurement instruments are expensive, difficult to install and maintain and less reliable than temperature measurement equipment. Therefore, heat flux estimation methods using temperature measurements are the preferred approach for several applications. An inverse heat conduction problem (IHCP) must be solved to estimate the unknown heat flux at the surface, knowing the temperature measurements at interior points of the medium. Several methods have been developed and applied for solving IHCPs [2]. Using ANN as an approach to solve IHCPs has also been studied in several articles. Most of these studies used the whole time domain data for training and applying the network. Raudensky et al. [3] used ANN for coupled parameter and function specification in IHCPs. Jambunathan et al. [4] used neural networks to determine the convective heat transfer coefficients. Krejsa et al. [5] presented a summary of efforts to achieve solution of the IHCP using ANNs for both whole time domain mapping and sequential mapping. They concluded that ANN can be used to solve IHCPs and show that using an adaptive linear network cannot account for the noise in the input data while the back propagation network can handle the noise better. Sablania et al. [6] presented ANN models for calculating the convective heat transfer coefficient at the surface of a cube and semi-infinite plate using temperature measurements inside the medium. Evaluation of heat flux distribution generated by a flame gun in a cylindrical coordinate system is studied by Hao et al. [7].

More attention has been attracted to approaches with real-time capability during recent years. Ijaz et al. [8] used a Kalman filter to solve a two-dimensional transient IHCP. They used the results from the adaptive estimator developed for transient heat flux estimation at the boundary in two-dimensional heat conduction domain with heated and insulated walls. Feng et al. [9] used Laplace transforms to relate the measured conditions at one end of a domain to the unknown conditions at the remote surface. ANN is also used for real-time heat flux estimation in few studies. A filter solution based on the idea of training neural networks is studied by Kowsari et al. [10]. Deng and Hwang [11] used neural network to compute the temperature distribution in forward heat conduction problems and solved inverse heat conduction problems using a back propagation neural network to identify the unknown boundary conditions. They reported that the proposed neural network analysis method can solve forward heat conduction problems and is capable of estimating the unknown parameters in inverse problems with acceptable error. Khorrami et al. [12] used a linear neural network for real-time estimation of multi-component heat flux.

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Nomenclature

b	bias
$\mathbf{F}$	filter matrix
I	identity matrix
K	thermal conductivity, W/(mK)
L	length, m
M	current time step
m_f	number of future time steps
m_p	number of previous time steps
$\mathbf{q}$	vector of heat flux, W/m^2
q_M	heat flux at M's the time step
$\mathbf{T}$	vector of estimated temperature, K
t	time, s
W	weights between neural network's layers
$\mathbf{X}$	matrix of sensitivity coefficients
$\mathbf{Y}$	vector of measured temperature
Greeks	
α	thermal diffusivity, m^2/s
α_0	Tikhonov regularization parameter
γ_m	eigenvalue for X22 problem
ϕ_i^n	transfer function used for the neural network

Digital filter representation is one of the methods which can be used for real-time heat flux estimation using available temperature measurements. The idea of the filter algorithm is that the solution for the heat flux at any time is only affected by the recent temperature history and a few future time steps. Woodbury and Beck [1] studied the structure of the Tikhonov regularization problem and concluded that the method can be interpreted as a sequential filter formulation for continuous processing of data. They show that the computed heat fluxes using the whole domain solution and the filter coefficient solution are virtually the same for the constant-property solutions. The idea of using digital filter representation method is further developed for IHCPs using measured temperature history as remote boundary condition [13] and in multi-layer domains [14–16].

The primary goal of this study is to use ANN as a digital filter method for real-time heat flux estimation and compare some similarities of this approach with the digital filter representation of the Tikhonov regularization method. ANN consists of a set of interconnected neurons that can evaluate outputs from inputs by feeding information through the network and adjusting the weights [17–19]. In this work, a one-dimensional slab, initially at zero temperature, and with temperature-independent properties, with the perfectly insulated boundary at one end and subject to a change in heat flux at the other end is considered (this case is called X22B-0T0 in Ref. [20]). The ANN is trained using a triangular heat flux input and corresponding temperatures. The trained network is tested afterwards through several test cases using exact solutions for different heat flux profiles.

The ANN approach is also tested for media with temperature dependent material properties and with the assumption of same boundary conditions mentioned above. One strength of the digital filter approach is the capability of handling non-linear problems by interpolating the filter coefficients [21]. It is shown that ANN can also be used to perform this task using both linear and non-linear transfer functions for the hidden layer.

2. Inverse heat conduction problem (IHCP)

A forward heat conduction problem is known as determining temperature through the domain when knowing the boundary conditions. In an inverse problem, on the other hand, the temperature data are given for one or more points within the domain and the active boundary condition is unknown.

In this paper, a 1D slab is assumed which is insulated at $x = L$ and a heat flux is applied at $x = 0$. A schematic of the problem is shown in Fig. 1. This is denoted as X22B-0T0 case in Cole et al. [20]. The mathematical statement of this problem can be given as:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \tag{1}$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = 0, \quad T(x,0) = T_0 \tag{2a, b}$$

The unknown boundary condition:

$$q(0,t) = -k\left.\frac{\partial T}{\partial x}\right|_{x=0} \tag{3}$$

A solution for the X22B10T0 case with a constant heat flux at $x = 0$ and zero heat flux at $x = L$ is [20]:

$$\frac{T_{X22}(x,t)}{q_c L/k} = \frac{\alpha t}{L^2} + \frac{1}{3} - \frac{x}{L} + \frac{x^2}{2L^2} - 2\sum_{m=1}^{m_{\max}} \frac{\cos(\gamma_m x/L)}{\gamma_m^2}\exp(-\gamma_m^2 \alpha t/L^2) \tag{4}$$

where $\gamma_m = m\pi$ is the mth eigenvalue ($m = 1,2,3,\ldots$).

The method employed in this section is the filter coefficients representation for Tikhonov regularization method. This method is discussed extensively in Ref. [1].

Knowing the temperature measurements at one point through the domain ($x = x_1$), the heat flux on the surface can be found using the following equation:

$$\hat{\mathbf{q}} = \mathbf{F}\mathbf{Y} \tag{5}$$

where $\mathbf{F}$ is the filter matrix and $\mathbf{Y}$ is the vector of temperature measurements at x_1. The filter matrix can be given as:

$$\mathbf{F} = (\mathbf{X}^{\mathbf{T}}\mathbf{X} + \alpha_0\mathbf{I})^{-\mathbf{1}}\mathbf{X}^{\mathbf{T}} \tag{6}$$

where $\mathbf{X}$ is the sensitivity matrix and α_0 is the Tikhonov regularization parameter. The filter matrix has several interesting characteristics [1]. All of the rows of the filter matrix have the same entries but are shifted in time, except for the first few terms and last few terms of each row. This structure is shown in the equation below:

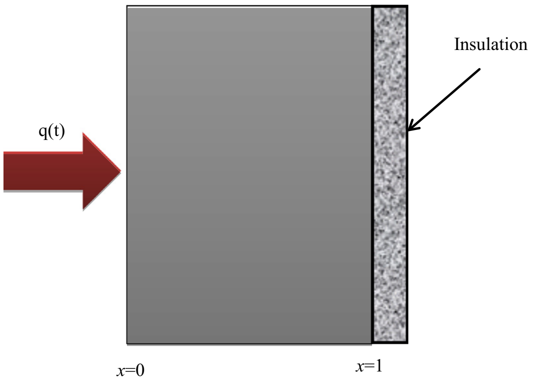

Fig. 1. Schematic of the problem.