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Numerical study of effective thermal conductivities of plain woven composites by unit cells of different sizes

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ABSTRACT

An FEM (Finite Element Method) numerical approach of predicting the effective thermal conductivities of plain woven composites is presented in this paper. Three reducing-size unit cells are formulated by using different symmetries exhibited in the composite, including translational, reflectional and rotational symmetries. Corresponding thermal boundary conditions are derived and validated by the numerical results of the same problem of different unit cells. Thermal conductivities of the matrix with porosity and the woven yarns are calculated first, and then used as input data to numerically predict the effective thermal conductivities of plain woven composite. The influences of porosity and fiber volume fraction on effective thermal conductivities of studied composites are clarified.

1. Introduction

For its low weight and high strength plain woven composite is widely used in aeronautics engineering, automobile industries and other related fields, and the prediction of its mechanical property has been a hot research topic during past decades [1–4]. With regard to the service environment, taking structure parts of aircraft as an example, plain woven composites may experience a complex and harsh mechanical and thermal test. Accurate predictions of the composites' thermal characteristics are of great importance for a reliable design in many engineering fields and have attracted researchers' attention [5–8]. Similar to other composites, the calculation of effective thermal conductivity of the plain woven composites can be conducted at two length scales, the meso-scale and macro-scale. The meso-scale corresponds to interwoven yarns and matrix, while the macro-scale corresponds to woven composites. The woven yarn can be considered as unidirectional fiber reinforced composites and presents transversely isotropic characteristic. Its axial thermal conductivity is often determined by so-called classic mixture rule, while the transverse thermal conductivity is usually numerically calculated [9,10]. To the authors' knowledge, studies that focus on the physical properties of the porous matrix of plain woven composites are very limited. Del et al. [11] classified and quantified porosities for some specific ceramic matrix composites (provided by the German Aerospace Research Establishment). Based on that, their numerical studies [5] took the influence of porosity into account, and the numerical results were validated by their experimental ones. Although these works are meaningful, they are not representative enough for the plain woven composites and more further works need to be done. On the other hand, for the study of porous matrix researches about composites with regular and random distributed second phase (particle or porosity) were conducted in [12–14], respectively.

At the macro-scale the plain woven composites can be studied based on the obtained properties of yarns and matrix. Analytical [1,2] and numerical method (FEM) [3,4] are two common ways to study the plain woven composites. For various composites with regular or irregular structures, most researches about the prediction of effective thermal conductivity of composites are often focused on a less scale element which can represent the larger scale composites [15–18]. The element is often called the unit cell or representative volume element (RVE). For textile reinforced composites such as plain woven composite, this representative element, i.e. unit cell can be formulated according to the composites' geometric symmetries.

According to Ref. [19], three types of symmetries can account for all symmetries in nature, i.e. reflection about a plane (reflectional symmetry), rotation about a axis by an angle (rotational symmetry) and translation along a axis (translational symmetry). Generally speaking, the use of different symmetries will formulate unit cells of different size and leads to boundary conditions of different complexities. For plain woven composites, a so-called *full*

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Nomenclature

Abbreviation

UC_1	the full unit cell
UC_2	the quarter unit cell
UC_3	the one-sixteenth unit cell

Symbols

A_i	area of the boundary plane perpendicular to i direction
a	length in x direction
b	length in y direction
C	stiffness matrix
c	height of woven yarns
h	length in z direction
M, M', M''	arbitrary node M and its corresponding nodes with symmetric transformations
O, O', O''	specific node O and its corresponding nodes with symmetric transformations
P_x, P_y	reflection plane $y = 0$, $x = 0$
Q_i	sum of the heat flow of all nodes on boundary plane perpendicular to i direction
$\boldsymbol{q}$, q_i	heat flux and its component in i direction
r	radius of spherical pores or fibers
S_x	spacings between two neighboring yarns in x directions
S_y	spacings between two neighboring yarns in y directions
V_f	fiber volume fraction of composite
V_y	yarn volume fraction of composite
V_{fy}	fiber volume fraction of woven yarns
u	the displacement in x direction
w	width of woven yarns
$\boldsymbol{\lambda}$	thermal conductivity matrix
λ_{SiC}	thermal conductivity of SiC
λ_{xx}, λ_{yy}, λ_{zz}	thermal conductivities in x, y, z directions
λ_{ya}	axial thermal conductivity of yarns
λ_{fa}	axial thermal conductivity of fibers
ΔT	temperature difference
$\nabla\boldsymbol{T}$, ∇T_i, ∇T_x, ∇T_y, ∇T_z	temperature gradient and its components in i, x, y, z directions
σ_i	stress component in i direction
ε_i	strain component in i direction

Subscripts

x_1, y_1, z_1	coordinates of nodes
x, y, z	x, y, z component directions

unit cell which corresponds to the full unit cell UC_1 of this paper, is often formulated with the adoption of translational symmetries only [3,20–22]. This leaves some room to fully employ the advantages of symmetries for further reduction of the size of the unit cell. Tanov and Tabiei [23] developed the so-called *representative volume cell* and *one quarter cell* and conducted stress analysis with available strain field without any derivation of the boundary conditions. These two cells, i.e. *representative volume cell* and *one quarter cell*, respectively correspond to the quarter unit cell UC_2 and the one-sixteenth unit cell UC_3 in this work and will be discussed later. Li et al. [24] have formulated the one-sixteenth cell. In their paper the utility of reflectional and rotational symmetries is described in detail, appropriate mechanical boundary conditions are derived rigorously according to corresponding symmetries, and the boundary conditions are validated by stress analysis. To the authors' knowledge, the thermal boundary conditions corresponding to the above three unit cells have not been discussed in references so far.

The major contents of the present study are as follows. First the corresponding thermal boundary conditions for three unit cells, i.e. the full cell UC_1, the quarter cell UC_2 and the one-sixteenth cell UC_3, are derived; Then numerical prediction of the effective thermal conductivities of the plain woven composites are implemented. The boundary conditions are validated by the almost identical numerical results of different unit cells. Third, the influence of fiber and porosity volume fraction on the thermal characteristics of the plain woven composites is studied. In the paper, it is assumed that pores only exist in the matrix and the yarn is considered as unidirectional fiber reinforced composite.

2. Relative temperature and heat flux relations between corresponding nodes for three symmetric transformations

Before the derivation of the thermal boundary conditions for the three unit cells, the relative temperature and heat flux relations between corresponding nodes of the three symmetric transformations should be specified first. Also it would be helpful to clarify two kinds of stimulus, the so-called symmetric thermal stimulus which is the heat flux parallel to the reflection plane or the rotation axis, and the so-called antisymmetric thermal stimulus which is the heat flux perpendicular to the reflection plane or the rotation axis. This idea comes from the analogical relations of heat flux vs. temperature gradient, and that of stress vs. strain as shown in Eq. (1). In the equation, σ_i, ε_i, q_i and ∇T_i are stress, strain, heat flux and temperature gradient components in i direction, respectively. $\mathbf{C}$ and $\boldsymbol{\lambda}$ are stiffness and thermal conductivities matrix, respectively. These two thermal stimuli respectively correspond to the symmetric and antisymmetric loadings in mechanics [19,24], as schematically presented in Fig. 1. In the figure, the thermal stimulus and mechanical loading are displayed. The relations of relative temperature between corresponding nodes and the relative displacement between corresponding nodes (which are the key relations used in the derivation of thermal and mechanical boundary

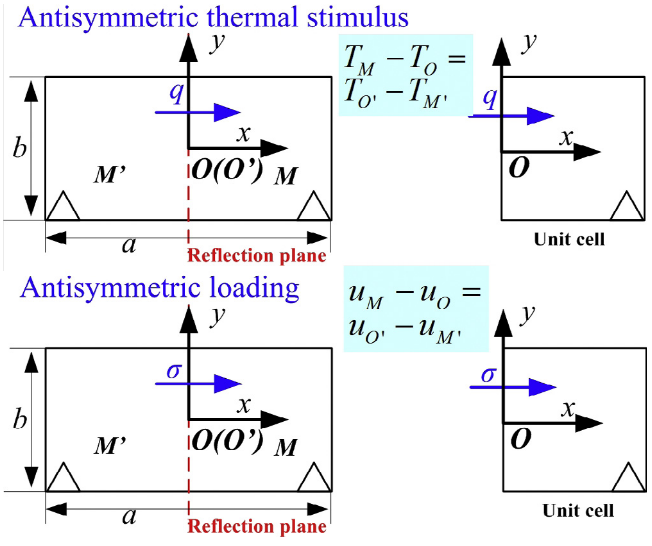

Fig. 1. Reflectional symmetry and the antisymmetric stimuli.