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On laminar natural convection inside multi-layered spherical shells

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ABSTRACT

Laminar natural convection flow inside multi-layered spherical shells with internal hot and external cold boundaries was investigated. Direct numerical simulations (DNS), which were performed by utilizing the immersed boundary method, addressed the fully 3D natural convection flow inside spherical shells with concentric, eccentric, equi-spaced and non-equi-spaced zero-thickness internal baffles. The insulation efficiency of the spherical shell was studied for up to four equi-spaced concentric internal layers. A unified functional dependency correlating modified Nu^* and Ra^* numbers was derived for spherical shells with up to four equi-spaced concentric internal layers. The effects of both vertical and horizontal eccentricity of the internal layers and of the width variation of concentric layers on the overall insulating performance of the spherical shell were investigated and quantified in terms of the *Nu–Ra* functionality.

1. Introduction

Buoyancy-driven flow developing inside spherical annuli has been the subject of considerable research, both theoretical and experimental for the past fifty years. Typically, the buoyancy-driven flow between two isothermal concentric spheres (where each sphere is held at a different temperature) has been investigated as a function of the diameter ratio, $\phi = D_i/D_o$, and the Rayleigh, *Ra*, and Prandtl, *Pr*, numbers. The pioneering experimental studies of Bishop et al. [1,2], which focused on visualization of the flow, indicated three distinct types of flow pattern – “crescent eddy”, “kidney-shaped” and “falling vortices” – that depend on the diameter ratio, ϕ, of the shells. Their experimental results were confirmed by the study of Mack and Hardee [3], who derived a low-Rayleigh-numbers analytical solution for the natural convection of air between two concentric spheres. More recently, the natural convection flow of working fluids other than air (namely, water and silicone oils) was experimentally addressed by Scanlan et al. [4] and visualized by Yin et al.[5]. The later group described naturally induced flow patterns and categorized the type of the flow for each fluid in terms of the inverse of the relative gap width and the Rayleigh number. Subsequent numerical studies on steady and transient natural convection flow inside spherical shells extended the state of the art to an even wider range of *Pr* ($0.71 \leqslant Pr \leqslant 100$) [6,7] and *Ra* ($10^2 \leqslant Ra \leqslant 5 \times 10^5$) [7] numbers and to the analysis of vertically eccentric configurations [8].

The theoretical analysis of unsteady natural convection inside a differentially heated spherical annulus is a challenging problem, since different flow regimes can dominate locally in its different regions, taking the form of Rayleigh-Bènard convection at the top of the shell, of a differentially heated cavity at the near-equatorial region, and of a thermally stable flow regime at the bottom of the shell. Moreover, instabilities and transition scenarios are sensitive to the value of the *Pr* number and to the ratio of the internal to external diameter ϕ [9,10]. For shells with an internal hot boundary and an external cold boundary, the flow patterns vary with the ratio ϕ: Powe et al. [11] described a “modified kidney shaped eddy” for wide shells ($\phi \leqslant 0.5$), an “interior e xpansion–contraction” for $0.5 \leqslant \phi \leq 0.65$, a “three dimensional spiral” flow for $0.65 \leqslant \phi \leq 0.85$, and a “falling vortices” pattern for narrow shells ($0.85 \leqslant \phi$). Futterer et al. [12] reported that the flow inside shells of large and moderate widths ($0.41 \leqslant \phi \leq 0.71$) with a cold internal boundary and a hot external boundary exhibited an unsteady “dripping blob” phenomenon for $Pr = \infty$.

Natural convection inside a spherical annulus comprises an essential heat transfer mechanism in various engineering design problems, such as in solar energy collectors, storage tanks, thermal energy storage (TES) systems and nuclear reactors. Another potential application of spherical annuli is related to the design of the Titan Montgolfiere hot air balloon, which was recently chosen by NASA as the air-robot vehicle of choice for the exploration of Titan’s atmosphere. Given Titan’s low gravity (one-seventh that of Earth) and its cryogenic atmospheric temperatures (72–94 K), heat transfer by radiation can safely be neglected, and natural convection can be regarded as the only heat transfer mechanism for the stationary suspended balloon. Such a balloon, designed to

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provide a minimized heat flux rate through its walls, could serve as a sustainable air-robot platform for carrying a payload sufficient for a long-term space mission.

The concept of the double-walled Titan Montgolfiere, for which the spherical shell plays the role of a thermal insulator separating the hot interior of the balloon from the cold surroundings, has recently been established and investigated by Samanta et al. [13] and Feldman et al. [14]. One of the main findings of both studies was that theoretical estimation of the heat flux rate through the boundaries of the insulating gap of both scaled and full-scale balloons has the greatest uncertainties. This finding motivated further research [15], which was focussed on a more detailed analysis of transitional and fully turbulent natural convection flows inside narrow spherical differentially heated shells ($0.8 \leqslant \phi \leq 0.9$) and yielded an improved Nusselt (Nu)-Ra correlation derived specifically for that range of ϕ values.

The current study is aimed at further developing high-fidelity computational fluid dynamics (CFD) concepts for minimizing the heat flux rate through an insulating gap of spherical shape. In particular, the natural convection flow inside multi-layered differentially heated spherical shells with internal baffles of zero thickness is studied by DNS. The flow developing inside spherical shells characterized by both equi-spaced/non-equi-spaced and concentric/eccentric distributions of the internal baffles is simulated. The immersed boundary method (IBM) is utilized for treating the internal and external shell walls. Additionally, a novel modified $Nu^* - Ra^*$ correlation is derived for a spherical shell with up to four internal equi-spaced concentric layers in the range of $10^3 \leqslant Ra \leqslant 10^7$.

2. Physical model and governing equations

The natural convection flow inside single- or multi-layered spherical shells formulated in Cartesian coordinates (x, y, z) with the origin located at the center of the shell and gravity acting opposite to the positive direction of z axis (see Fig. 1) is governed by the following non-dimensional Navier–Stokes (NS) and energy equations:

$$\nabla \cdot \boldsymbol{u} = 0 \quad (1)$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \sqrt{\frac{Pr}{Ra}}\nabla^2 \boldsymbol{u} + \theta \vec{e}_z \quad (2)$$

$$\frac{\partial \theta}{\partial t} + (\boldsymbol{u} \cdot \nabla)\theta = \frac{1}{\sqrt{PrRa}}\nabla^2 \theta, \quad (3)$$

where $\boldsymbol{u} = (u,v,w)$, p, t, and θ are the non-dimensional velocity, pressure, time and temperature, respectively, and $\vec{e}_z$ is a unit vector in the vertical (z) direction. The Boussinesq approximation $\rho = \rho_0(1 - \beta(T - T_c))$ was applied to address the flow buoyancy effects. As a result, an additional temperature term appears as a source in the momentum equation in the z direction (see Eq. (2)), thereby allowing for the temperature–velocity coupling. The problem is scaled by L, $U = \sqrt{g\beta L\Delta T}$, $t = L/U$, and $P = \rho U^2$ for length, velocity, time, and pressure, respectively. Here, $L = R_o - R_i$ is the total shell width, defined as a difference between the outer, R_o and the inner R_i radius of the shell, ρ is the mass density, g is the gravitational acceleration, β is the isobaric coefficient of thermal expansion, and $\Delta T = T_h - T_c$ is the temperature difference between the hot and cold boundaries. The non-dimensional temperature θ is defined as $\theta = (T - T_c)/\Delta T$. The Ra and Pr numbers are $Ra = \frac{g\beta}{\nu\alpha}\Delta T L^3$ and $Pr = \nu/\alpha$, where ν is the kinematic viscosity and α is the thermal diffusivity. All the simulations were performed for the value of $Pr = 0.71$ corresponding to air.

The IBM [16] was implemented for imposing Dirichlet boundary conditions for the temperature and velocity fields at the spherical shell boundaries and the internal baffles. The IBM is not a standalone solver; rather, it requires a "driver" with which to be combined and its implementation should be perceived as a philosophy of enforcing boundary conditions. In principle, such a "driver" can be any time-marching solver, whose efficiency is typically boosted by choosing a computational domain of rectangular/prismatic shape and by utilizing a structured grid for spatial discretization of the NS and energy equations. In the present formulation the flow within the differentially heated spherical shell is an integral part of a more general natural convection flow within the whole cube, including also the outer ($R > R_o$) and the inner ($R < R_i$) regions (see Fig. 1). The flow was simulated by applying no-slip boundary conditions at all the cube faces, which were held at a constant temperature T_c (the same as the temperature of external boundary of the spherical shell). In the following, only the results relevant to the spherical shell region are discussed. Below, we detail the IBM formulation implemented in the present study.

Fig. 2 shows the setup of a typical spatial discretization implemented on a staggered grid. The grid is characterized by offset velocity, temperature and pressure fields. An immersed object of

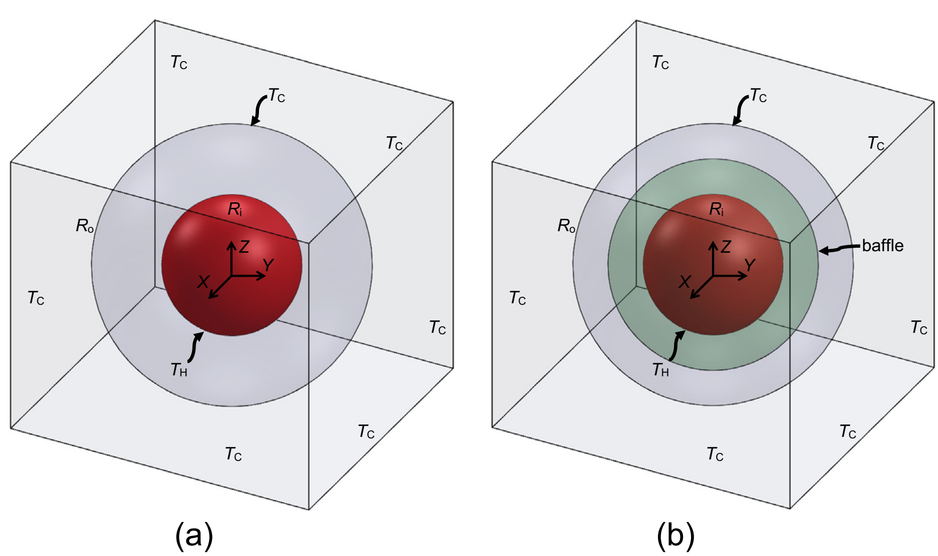

Fig. 1. Physical model and system of coordinates for the spherical shell: (a) with no internal baffles; (b) with a single internal baffle and two concentric equi-spaced layers.