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A new inverse analysis method based on a relaxation factor optimization technique for solving transient nonlinear inverse heat conduction problems

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A B S T R A C T

The relaxation factor is a key parameter in gradient-based inversion and optimization methods, as well as in solving nonlinear equations using iterative techniques. In gradient-based inversion methods, the relaxation factor directly affects the inversion efficiency and the convergence stability. In general, the bigger the relaxation factor is, the faster the inversion process is. However, divergences may occur if the relaxation factor is too big. Therefore, there should be an optimal value of the relaxation factor at each iteration, guaranteeing a high inversion efficiency and a good convergence stability. In the present work, an optimization technique is proposed, using which the relaxation factor is adaptively updated at each iteration, rather than a constant during the whole iteration process. Based on this, a new inverse analysis method is developed for solving multi-dimensional transient nonlinear inverse heat conduction problems. One- and two-dimensional transient nonlinear inverse heat conduction problems are involved, and the instability issues occurred in the previous works are reconsidered. The results show that the new inverse analysis method in the present work has the same high accuracy, the same good robustness, and a higher inversion efficiency, compared with the previous least-squares method. Most importantly, the new method is more stable by innovatively optimizing and adaptively updating the relaxation factor at each iteration.

1. Introduction

Inverse heat conduction problems (IHCPs) are frequently encountered in various important engineering applications [1,2]. IHCPs are ill-posed/conditioned, which are very challenging, because small disturbances of input conditions or measurement errors can cause large solution errors and oscillations. The objective of IHCPs is to identify the boundary or initial conditions, thermo-physical properties or geometric parameters by using additional information such as temperature measurements within the domain or on the boundary [3]. Transient states and temperature-dependent thermo-physical properties are usually involved in IHCPs. Consequently, the solution of a transient nonlinear inverse heat conduction problem is difficult, especially if multi-dimensions and measurement errors are concerned. IHCPs have been extensively studied [3–14] and many methods have been proposed [15–21].

Generally, inversion methods can be classified into two categories: one is the gradient-based method and the other is the stochastic method [3,22]. The advantage of the stochastic methods is their capability in searching for the global optimum. However, these methods usually require a large number of iterations [23]. The advantages of the gradient-based methods are the fast convergence speed and the high accuracy. In gradient-based methods, the determination of sensitivity coefficients [3,23] is the main work. However, sensitivity coefficients are difficult to be precisely calculated if multi-dimensions, transient states and nonlinearities are involved, and this is the main reason why so many stochastic methods have been developed.

In recent years, the first author and the co-authors have been focused on the accurate calculation of sensitivity coefficients in gradient-based methods [3,23–27]. We introduced the com plex-variable-differentiation method (CVDM) [28] into inverse heat transfer problems, and have successfully overcome the difficulty in accurately calculating sensitivity coefficients. Based on CVDM, a least-squares method was developed, and numerical examples showed that it was efficient, accurate and robust [23–27].

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Nomenclature

a	coefficient to be recovered in Eq. (30), W/m^2
b	coefficient to be recovered in Eq. (30), 1/s
c	heat capacity, J/(kg K)
d	coefficient to be recovered in Eq. (30)
E_{rms}	root mean square deviation between the recovered/inverted and the exact/real values
$f(X)$	real function with variable X
$f(X+ih)$	complex function with real variable X and imaginary part h, to be expanded into Taylor series
F	dimensionless objective function
h	imaginary part in complex variable
K	the iteration number
L	the length of the modeled object in x-direction, m
M	total number of measured temperatures
N	total number of inverted parameters
q	heat flux, W/m^2
R	residual vector, K
t	temperature, K
W	the width of the modeled object in y-direction, m
w	relaxation factor
X	real variable in function f
x	x-coordinate, m
y	y-coordinate, m
z	vector of inverted parameters

Greek

γ	exact value
Δ	change in variable
ζ	random measurement error
η	random number
λ	thermal conductivity, W/(m K)
ξ	small positive number
ρ	density, kg/m^3
χ	recovered/inverted value
τ	heating time, s

Subscripts

0	initial time
b	bottom
exact	exact
i	the ith component of a vector
l	left
r	right
u	upper

Superscripts

0	initial guess
*	measured

In the previous works [23–27], the relaxation factor was a key parameter between 0 and 1, and it was a constant during the whole iteration process, which was empirically determined by trials. The relaxation factor directly affects the inversion efficiency and the convergence stability. In general, the bigger the relaxation factor is, the faster the inversion process is [26]. However, divergences may occur if the relaxation factor is too big [26]. We adopted dimensionless objective function [23] to overcome this drawback, and the convergence stability was improved. Unfortunately, divergences would occur again if the relaxation factor became bigger enough, under some circumstances, and the value of the relaxation factor was still empirically determined by trials.

Through the above analysis, the relaxation factor should not be too small for a fast iteration, while it should not be too big for a convergence stability. Therefore, there should be an optimal value of the relaxation factor at each iteration, guaranteeing a high inversion efficiency and a good convergence stability. This is the main idea that the present work stems from.

In the present work, a relaxation factor optimization technique is proposed, based on which a new inverse analysis method is developed. In the new method, the least-squares method in the previous works [23–27] is employed, and the relaxation factor is adaptively updated at each iteration, rather than an empirical constant during the whole iteration process. The advantages of high efficiency, good accuracy and robustness of the least-squares method are to be kept, and the convergence stability is expected to be improved, to effectively resist the ill-posedness.

The relaxation factor is a key parameter in gradient-based inversion and optimization methods, as well as in solving nonlinear equations using iterative techniques. To the best knowledge of the authors, it is the first study in optimizing the relaxation factor. This would be a major contribution of the present work.

Multidimensional transient nonlinear inverse heat conduction problems are involved. The instability issues occurred in one- and two-dimensional transient nonlinear inverse heat conduction problems in the previous works [23,26] are reconsidered.

2. Multidimensional transient nonlinear inverse heat conduction problems

2.1. One-dimensional transient nonlinear heat conduction problem

Consider a one-dimensional transient nonlinear heat conduction problem, in which the source term and the phase change are not involved. The heat conduction problem with temperature-dependent thermo-physical properties can be written as follows:

$$\rho(t)c(t)\frac{\partial t(x,\tau)}{\partial \tau} = \frac{\partial}{\partial x}\left[\lambda(t)\frac{\partial t(x,\tau)}{\partial x}\right] \tag{1}$$

with the initial condition given in Eq. (2).

$$t(x,\tau)|_{\tau=0} = t(x) \tag{2}$$

The boundary conditions in Eq. (3) are considered.

$$\begin{cases} \lambda(t)\frac{\partial t(x,\tau)}{\partial x}|_{x=L} = q_u(\tau) \\ t(x,\tau)|_{x=0} = t_b(\tau) \end{cases} \tag{3}$$

2.2. Two-dimensional transient nonlinear heat conduction problem

The two-dimensional transient nonlinear heat conduction problem with temperature-dependent thermo-physical properties can be written as follows:

$$\rho(t)c(t)\frac{\partial t(x,y,\tau)}{\partial \tau} = \frac{\partial}{\partial x}\left[\lambda(t)\frac{\partial t(x,y,\tau)}{\partial x}\right] + \frac{\partial}{\partial y}\left[\lambda(t)\frac{\partial t(x,y,\tau)}{\partial y}\right] \tag{4}$$

with the initial condition given in Eq. (5).

$$t(x,y,\tau)|_{\tau=0} = t(x,y) \tag{5}$$

and the Neumann boundary conditions given as

$$\lambda(t)\frac{\partial t(x,y,\tau)}{\partial y}\bigg|_{y=W} = q_u(x,\tau) \tag{6}$$