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# Analytic equations for the Wilson point in high-pressure steam flow through a nozzle

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ABSTRACT

An analytical method for the position and flow properties of Wilson point in high-pressure steam flow through a nozzle was presented. The thermodynamic properties of real gas and essential empirical equations at high-pressure for both superheated and supercooled steam were used to solve the position of Wilson point. Firstly, the analytic equation for the isentropic flow of real gas before Wilson point was built and a two dimensional bisection method was utilized for solving this nonlinear equation. And then, combining Rayleigh flow relationships, the position and properties of Wilson point can be efficiently solved by only giving the values of the nozzle geometry, inlet stagnation pressure and temperature. The algebraic solutions are well in agreement with high-pressure experimental data by Bakhtar. The relative deviations between calculation results and experimental data for three type nozzles are 0.09–0.56%, 0.09–0.57%, and 0.56–1.20% respectively. At last, a serial of isobaric and isothermal lines for the position of Wilson point were obtained to illustrate the power of this method. This analytic method contributes to the further research of theory, simulation and experiment at high-pressure steam flow.


## 1. Introduction

An analytical solution for the position and flow properties of Wilson point in nozzle flow at low-pressure (LP) was presented in our previous paper [1], which is valid just for ideal-gas flow and only could be extended to solution of the Wilson point in the LP flow devices. However, with the advent of high-pressure turbine [2] and the water-cooled nuclear reactor [3,4], the requirement has existed for the investigations to be extended to high-pressure (HP).

Because the condensation process in the most flow devices are very complicated, a variety of converging–diverging nozzles which have simple structure are always been chosen for numerical and experimental research [1]. A lot of results for the flow behavior of condensation phenomena in nozzles flow at high-pressure were obtained. The process of nucleation in high-pressure steam was described satisfactorily by one-dimensional numerical model and an experiment was carried out to measure the droplet size and thermodynamic properties of steam expanding through a convergent–divergent nozzle reported by Cinar [5]. Bakhtar [3,6,7] proposed one-dimensional steady-state numerical model for high-pressure superheated and supercooled steam through a nozzle and the calculation results had a good agree well with the experimental data. To improve the accuracy of numerical model, Gerber [8], Wróblewski [9], and Dykas [10] have put forward two-dimensional condensation models for high-pressure steam flow through a nozzle respectively.

However, the drawback numerical method is that the computational problem requires a sufficient number of the grid number, computing resources and processing time to keep track of the growing droplets in the complicated multi-dimensional flow field. Worse, it is unable to obtain the dominant physical effects with many interacting phenomena and in the nucleation process [1,11]. It is expected to achieve an algebraic expression about the nucleation process, especially the flow properties at the Wilson point. Dobbins [11], Huang [12], Gyarmathy [13], Clarke [14], Delale [15], and Ding [1] presented respectively an analytic method for determining the Wilson point at low-pressure. However, there is no an effective method that is suitable to high-pressure steam flow.

In this study, an accurate analytical method for solving the position and flow properties of the Wilson point in high-pressure steam flow through a nozzle was presented. Certainly, although the following analysis focuses only on nozzle flow, this method can be extended to the solution of the Wilson point in LP/HP turbines and every other flow device at high-pressure.

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**Nomenclature**

| | |
|---|---|
| $A$ | area, $m^2$ |
| $a$ | Helmholtz energy, $J\,kg^{-1}$ |
| $c_v$ | isochoric heat capacity, $J\,kg^{-1}\,K^{-1}$ |
| $c_p$ | isobaric heat capacity, $J\,kg^{-1}\,K^{-1}$ |
| $d$ | throat diameter of nozzle, m |
| $f_p$ | expansion rate coefficient, – |
| $H$ | the depth of nozzle, m |
| $h$ | specific enthalpy, $J\,kg^{-1}$ |
| $h_{fg}$ | latent heat of water vapor, $J\,kg^{-1}$ |
| $I$ | nucleation rate, $kg^{-1}\,s^{-1}$ |
| $K$ | Boltzmann's constant. $1.38\times10^{-23}\,J\,K^{-1}$ |
| $k$ | isentropic exponent, $Xc_p/c_v$, – |
| $k_p$ | expansion rate, $s^{-1}$ |
| $M$ | Mach number, – |
| $m_m$ | mass of water molecule, $2.99\times10^{-26}$ kg |
| $p$ | pressure, Pa |
| $p_{cr}$ | critical pressure, $2.2088\times10^7$ Pa |
| $R$ | specific gas constant, $J\,kg^{-1}\,K^{-1}$ |
| $r$ | droplet radius, m |
| $s$ | specific entropy, $J\,kg^{-1}\,K^{-1}$ |
| $T$ | temperature, K |
| $T_c$ | characteristic temperature, K |
| $T_{cr}$ | critical temperature, 647.286 K |
| $T_R$ | $T/T_{cr}$, reduced temperature of vapor, – |
| $u$ | internal energy, $J\,kg^{-1}$ |
| $V$ | velocity, $m\,s^{-1}$ |
| $v$ | specific volume, $m^3\,kg^{-1}$ |
| $w$ | sonic speed, $m\,s^{-1}$ |
| $X$ | function defined by Eq. (2) |
| $x$ | axial position of nozzle, m |
| $Y$ | function defined by Eq. (3) |
| $y$ | wetness fraction, – |
| $Z$ | compressibility factor, $Z(\rho, T)$, – |

*Greek*

| | |
|---|---|
| $\alpha$ | divergence half-angle of nozzle, ° |
| $\Delta T$ | subcooling, K |
| $\zeta$ | nucleation time coefficient, – |
| $\lambda$ | dimensionless velocity, – |
| $\rho$ | density, $kg\,m^{-3}$ |
| $\sigma$ | liquid surface tension, $N\,m^{-1}$ |
| $\tau_n$ | nucleation pulse duration, s |

*Subscripts*

| | |
|---|---|
| 0 | stagnation condition at inlet |
| $i$ | isentropic flow |
| $l$ | liquid |
| $mix$ | mixture phase |
| $s$ | saturation |
| $v$ | vapor |
| $W$ | Wilson point |

## 2. Thermodynamic and condensation properties

The pure steam without impurities will occur homogeneous condensation only when it becomes supercooled across saturation line and reaches the Wilson point. The supercooled properties and nucleation model of the vapor at high-pressure are different from the known relation for the ideal gas.

### 2.1. Thermodynamic properties of supercooled steam

The evaluation of supercooled properties at the metastable region was based on the thermodynamic database for steam by Vukalovich [16] and might be written as

$$p = \rho_v RT_v(1 + B_1\rho_v + B_2\rho_v^2 + B_3\rho_v^3) = Z(\rho_v, T_v)\rho_v RT_v \tag{1}$$

This database utilizes an equation-of-state (EOS) based on a series of virial coefficients appearing as functions of temperature only. The expressions describing the virial terms are given in Appendix A.1.

Define dimensionless parameters $X$ and $Y$ as

$$X = \frac{\rho_v}{p}\left(\frac{\partial p}{\partial \rho_v}\right)_{T_v} = \frac{Z_{\rho_v}}{Z} \tag{2}$$

$$Y = \frac{T_v}{p}\left(\frac{\partial p}{\partial T_v}\right)_{\rho_v} = \frac{Z_{T_v}}{Z} \tag{3}$$

where,

$$\begin{cases} Z_{\rho_v} = Z + \rho_v\left(\frac{\partial Z}{\partial \rho_v}\right)_{T_v} \\ Z_{T_v} = Z + T_v\left(\frac{\partial Z}{\partial T_v}\right)_{\rho_v} \end{cases} \tag{4}$$

Combining with Eqs. (1)–(3), the expressions for $X$ and $Y$ become

$$X = 1 + \frac{\rho_v}{Z}\left(\frac{\partial Z}{\partial \rho_v}\right)_{T_v} = 1 + \frac{B_1\rho_v + 2B_2\rho_v^2 + 3B_3\rho_v^3}{1 + B_1\rho_v + B_2\rho_v^2 + B_3\rho_v^3} \tag{5}$$

$$Y = 1 + \frac{T_v}{Z}\left(\frac{\partial Z}{\partial T_v}\right)_{\rho_v} = 1 + \frac{T_v}{1 + B_1\rho_v + B_2\rho_v^2 + B_3\rho_v^3} \times \left(\rho_v\frac{dB_1}{dT_v} + \rho_v^2\frac{dB_2}{dT_v} + \rho_v^3\frac{dB_3}{dT_v}\right) \tag{6}$$

In addition, the differential form of Eq. (1) is as follows:

$$Z\frac{dp}{p} - Z_{\rho_v}\frac{d\rho_v}{\rho_v} - Z_{T_v}\frac{dT_v}{T_v} = 0 \tag{7}$$

Hence,

$$\frac{dp}{p} - X\frac{d\rho_v}{\rho_v} - Y\frac{dT_v}{T_v} = 0 \tag{8}$$

### 2.2. Nucleation and droplet growth

According to the classical nucleation theory, the nucleation rate may be written as [8]

$$I = C\sqrt{\frac{2\sigma}{\pi m^3}}\frac{\rho_v}{\rho_l}\exp\left(-\frac{4\pi r_c^2\sigma}{3KT_v}\right) \tag{9}$$

where $C$ is non-isothermal correction factor and is given by

$$C = \left(1 + 2\frac{\gamma - 1}{\gamma + 1}\frac{h_{fg}}{RT_v}\left(\frac{h_{fg}}{RT_v} - \frac{1}{2}\right)\right)^{-1} \tag{10}$$

It is assumed that $\rho_l \gg \rho_v$, $\rho_l - \rho_v \approx \rho_l$. The critical droplet radius $r_c$ for the virial equation of state is calculated as follows: