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The influence of viscous dissipation on the unsteady conjugate forced convection heat transfer from a fluid sphere

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ABSTRACT

The unsteady, conjugate, forced convection heat transfer from a fluid sphere to a surrounding fluid flow in the presence of viscous dissipation has been analysed. The fluid flow inside and outside the sphere was considered laminar, axisymmetric, steady and incompressible. The heat balance equations were solved numerically in spherical coordinates system by a splitting finite difference method. The influence of the Reynolds, Brinkman and Peclet numbers on the heat transfer mechanism and rate was analysed for different values of the physical properties ratios (thermal conductivity ratio, heat capacity ratio, viscosity ratio and density ratio).

1. Introduction

Viscous dissipation (heating) describes the degradation of mechanical energy into thermal energy. This phenomenon occurs in all flow systems. However, for most flow problems, the effect of viscous dissipation is considered negligible. It is important only for systems with large viscosity and large velocity gradients.

The influence of the viscous dissipation on the heat transfer was recently analysed for the following laminar flow problems:

(i) heat transfer in micro-heat-sinks for regular fluids flow [1,2] or for nanofluids flow [3,4];
(ii) heat transfer for laminar slip flow in micro-tubes, [5–9];
(iii) heat transfer in cavities with an inner rotating cylinder, [10];
(iv) thermo-convective instability in regular fluids or fluid-saturated porous medium, [11] (and the references cited herein).

For flow in fluid-saturated porous media different approaches were developed for the viscous dissipation function, [12–14].

The results presented in [1–11] (and the references cited herein) show that viscous dissipation is a strong function of the geometry of the system, Reynolds number, Brinkman number, Prandtl (or Peclet) number, Knudsen number (only for problem (ii) – the Knudsen number refers to the degree of rarefaction) and the Gebhart number (only for problem (iv)). The conclusion drawn in [1–11] is: ignoring viscous dissipation could affect accurate flow simulations and measurements.

The influence of viscous dissipation on the conjugate heat transfer from a sphere to a surrounding fluid flow was not analysed. Viscous dissipation is a source term for the energy balance equation. Until now, the chemical reaction was the only source/sink term considered in the analysis of the conjugate mass/heat transfer from a sphere to a surrounding fluid flow, [15–23]. The chemical reaction takes place in the surrounding fluid (external chemical reaction) [16,17,19] or inside the sphere (internal chemical reaction) [15,18–23]. The following types of chemical reaction were analysed:

- first-order irreversible, isothermal, [15–17,19];
- first-order irreversible, non-isothermal, [18];
- second-order irreversible, isothermal, [20,22,23];
- consecutive, second-order, isothermal, [21].

The results presented in [15–23] show that the conjugate heat/mass transfer from a sphere to a surrounding fluid flow in the presence of a chemical reaction depends on the chemical reaction strength, type and location.

The aim of the present work is to analyse the influence of the viscous dissipation on the unsteady, forced convection, conjugate heat transfer from a fluid sphere to a surrounding fluid flow. This problem was not investigated until now. Compared to the chemical reaction, viscous dissipation is a source term that does not depend on the process variables (concentration and/or temperature). The present computations are focused on the influence of the flow regime (Reynolds number), Brinkman and Peclet numbers on the

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Nomenclature

a	radius of the sphere
Br	Brinkman number, $Br = \mu U_0^2/k\,(T_{1,0} - T_{2,0})$
c_P	heat capacity
d	diameter of the sphere, $d = 2a$
k	thermal conductivity
Nu	instantaneous average Nusselt number
Nu_θ	instantaneous local Nusselt number
Pe	Peclet number, $Pe = U_0 d\rho c_P/k$
Pr	Prandtl number, $\mu c_P/k$
r	dimensionless radial coordinate, r^*/a, in spherical coordinate system
r^*	radial coordinate in spherical coordinate system
Re	Reynolds number, $U_0 d\rho/\mu$
t	time
T	temperature
U_0	velocity far away from the sphere
V_R	dimensionless radial velocity component
V_θ	dimensionless tangential velocity component
Z	dimensionless temperature defined by the relation, $Z_{2(1)} = \frac{T_{2(1)} - T_{2,0}}{T_{1,0} - T_{2,0}}$

Greek symbols

α	thermal diffusivity
Φ	thermal conductivity ratio, k_1/k_2
μ	dynamic viscosity
θ	polar angle in spherical coordinate system
κ	dynamic viscosity ratio, μ_1/μ_2
ρ	density
τ	dimensionless time or Fourier number, $\tau = 4t\alpha/d^2$
χ	density ratio, ρ_1/ρ_2
ψ	dimensionless stream function
Ξ	heat capacity ratio, $(\rho_1 c_{P,1})/(\rho_2 c_{P,2})$

Subscripts

1	refers to the interior of the sphere
2	refers to the exterior of the sphere
st	steady state value
0	initial conditions
∞	refers to the outside boundary condition

heat transfer mechanism and rate for different values of the physical properties ratios (thermal conductivity ratio, heat capacity ratio, viscosity ratio and density ratio).

This paper is organised as follows. In Section 2 we describe the mathematical model of the problem. Section 3 presents the numerical algorithm. The numerical experiments made and the results obtained are presented in Section 4. Finally, some concluding remarks are briefly mentioned in Section 5.

2. Model equations

Consider the laminar, viscous, steady, axisymmetric, incompressible flow of a Newtonian fluid with a superficial velocity U_0 and initial temperature $T_{2,0}$ past a fluid sphere with initial temperature $T_{1,0}$. Inside the sphere the flow is also laminar, steady, incompressible, axisymmetric, viscous and the fluid is Newtonian. The following additional assumptions are considered valid:

- during the heat transfer process, the volume and shape of the sphere remains constant;
- the effects of buoyancy and the work done by pressure changes are negligible;
- the physical properties are uniform, isotropic and constant;
- no emission or absorption of radiant energy;
- no phase change;
- no chemical reaction inside the sphere or in the surrounding medium.

For the scenario presented previously, the conjugate heat transfer is governed by the following dimensionless convection–diffusion equations, [24]:

$$\frac{\partial Z_i}{\partial \tau_i} + \frac{Pe_i}{2}\left(V_{R,i}\frac{\partial Z_i}{\partial r} + \frac{V_{\theta,i}}{r}\frac{\partial Z_i}{\partial \theta}\right) = \Delta Z_i + Br_i F_i, \tag{1}$$

where

$$\Delta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right),$$

$$V_{R,i} = \frac{1}{r^2\sin\theta}\frac{\partial\psi_i}{\partial\theta}, \quad V_{\theta,i} = -\frac{1}{r\sin\theta}\frac{\partial\psi_i}{\partial r},$$

$$Br_i = \frac{\mu_i U_0^2}{k_i(T_{1,0} - T_{2,0})}, \quad Pe_i = \frac{U_0 d\rho_i c_{P,i}}{k_i},$$

$$F_i = 2\left[\left(\frac{\partial V_{R,i}}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial V_{\theta,i}}{\partial\theta} + \frac{V_{R,i}}{r}\right)^2 + \left(\frac{V_{R,i}}{r} + \frac{V_{\theta,i}\cot\theta}{r}\right)^2\right] + \left(\frac{\partial V_{\theta,i}}{\partial r} - \frac{V_{\theta,i}}{r} + \frac{1}{r}\frac{\partial V_{R,i}}{\partial\theta}\right)^2,$$

$i = 1$ for the interior of the sphere ($r < 1$) and $i = 2$ for the exterior of the sphere ($r > 1$).

The boundary conditions are:

- axis of symmetry, $\theta = 0, \pi$;

$$\frac{\partial Z_i}{\partial\theta} = 0, \quad i = 1,\ 2, \tag{2a}$$

- surface of the sphere, $r = 1$;

$$Z_1 = Z_2, \quad \Phi\frac{\partial Z_1}{\partial r} = \frac{\partial Z_2}{\partial r}, \tag{2b}$$

- centre of the sphere, $r = 0$;

$$Z_1 = \text{finite}, \tag{2c}$$

- free stream, $r \to \infty$ ($r = r_\infty$);

$$Z_2 = 0. \tag{2d}$$

The dimensionless initial conditions are:

$$\tau_{1(2)} = 0, \quad Z_1 = 1, \quad Z_2 = 0. \tag{3}$$

When the viscosity ratio is not equal to one, the viscous dissipation function is discontinuous across the interface. According to Slattery [25], "*In the context of energy transfer, the most common interfacial phenomena arise either from the temperature dependence of the interfacial tension or from phase changes*". From the assumptions considered valid in this work, the following statements can be made:

- there is no mass transfer through the interface;