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# Constructal entransy dissipation rate minimization for solid–gas reactors with heat and mass transfer in a disc-shaped body

Huijun Feng, Lingen Chen *, Zhihui Xie, Fengrui Sun

*Institute of Thermal Science and Power Engineering, Naval University of Engineering, Wuhan 430033, PR China*
*Military Key Laboratory for Naval Ship Power Engineering, Naval University of Engineering, Wuhan 430033, PR China*
*College of Power Engineering, Naval University of Engineering, Wuhan 430033, PR China*

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ABSTRACT

A heat and mass transfer process for solid–gas reactors in a disc-shaped body is investigated in this paper. Radial- and branched-pattern integrated collectors are embedded in the body to enhance heat and mass transfers. The distributions of the integrated collectors in the two pattern discs are optimized using constructal theory and entransy theory. The results indicate that the optimal elemental volume fractions obtained by the minimizations of entransy dissipation rate and entropy generation rate are evidently different. There exists a critical radius, which decides whether the radial- or branched-pattern design is used. The decrease in the thermal conductivity ratio and increase in the first order volume fraction of the integrated collector lead to a better performance of the solid–gas reactor. The model with pure heat conduction or mass transfer is special case of the model in this paper. The results obtained in this paper can provide some design guidelines for the solid–gas reactors.

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## 1. Introduction

Heat or mass transfer commonly exists in various transfer systems. To improve their transfer performances are of great importance. Constructal theory [1–11] is one of the important theories for the performance optimizations of various transfer problems, such as "volume-point" [12–20] and "disc-point" [21–25] heat conduction problems, mass transfer problems in porous media [26–31], etc.

The optimizations of the heat and mass transfer systems [32–38] can be also accomplished with the help of constructal theory. Azoumah et al. [32,33] built a rectangular solid–gas reactor (SGR) model with heat conduction and gas diffuser, and optimized the structure of the model with minimum entropy generation rate (EGR) objective. They concluded that the constructal reactors generated less entropy than that of the elemental one, but the decrement of the entropy generation was not evident when the level of the SGR was higher than one. Zhou et al. [34] further built a triangular model of the SGR, and reduced the EGR of the model evidently compared with that of the rectangular one. Neveu et al. [35] separated the conductive and diffusive paths of the model in Ref. [32], and optimized its structure with minimum exergy impedance objective. They concluded that the volumes of the elemental reactor and first order construct were reduced by 33.33% and 66.67%, respectively, compared with those of the corresponding existing devices. Azoumah et al. [36] further built a more actual heat and mass transfer model in transient state, and obtained the maximum ratio of the power output to the entropy generation. They concluded that the first order construct had a better second law performance but a lower power output than the elemental one. Feng et al. [37] built a disc-shaped model of the SGR, and optimized it based on EGR minimization. They concluded that the EGR of the SGR did not increase with its complexity, but depended on the relationship of the critical radius and disc radius. Moreover, the constructal design was also introduced into the optimizations of the solar thermochemical reactors [38].

To reveal the essential property of a heat transfer process, Guo et al. [39,40] proposed a new physical quantity–"entransy", $E_{vh} = \frac{1}{2}Q_{vh}U_h = \frac{1}{2}Q_{vh}T$, where $Q_{vh}(= Mc_vT)$ and $U_h$ $(T)$ are the thermal capacity with constant volume and the thermal potential, respectively. Henceforth, the heat transfer ability of an object could be described by "entransy". Based on entransy concept, the extremum principle of entransy dissipation for heat conduction process was further proposed [39,40]. Based on this principle, some scholars have performed various optimizations for heat or

* Corresponding author at: College of Power Engineering, Naval University of Engineering, Wuhan 430033, PR China. Tel.: +86 27 83615046; fax: +86 27 83638709.
*E-mail addresses:* lgchenna@yahoo.com, lingenchen@hotmail.com (L. Chen).

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mass transfer processes [15,20,22,41–55] and thermodynamic cycles [43,44,56–60], which greatly enriches the connotation of entransy theory.

The "disc-point" models were introduced into the optimizaitions of the heat and mass transfer problems [21–25,37], and the better performances were exhibited in these models. The EGR was taken as the optimization objective in the constructural design of the disc-shaped SGR [37]. The optimizations of transfer processes based on entransy theory have provided various useful guidelines for the designs of the transfer systems [15,20,22,41–60]. Consider these situations, the "disc-point" model of the SGR with heat conduction and gas diffuser in Ref. [37] will be re-investigated in this paper by taking the minimization of the entransy dissipation rate (EDR) as optimization objective. The entransy theory and constructal theory will be introduced to optimize the constructs of the SGRs. A comparison between the optimal constructs of the SGRs obtained by the minimizations of the EDR in this paper and the EGR in Ref. [37] will be performed.

## 2. Radial-pattern disc of the solid–gas reactor

Consider a disc-shaped solid–gas reactor (SGR) combined heat and mass transfer as shown in Fig. 1. It comprises the reactive porous material (radius $R_0$, thermal conductivity $k_0$, and permeability $K_0$) and a number ($N$) of radial-pattern integrated collectors (length $R_0$ and width $D_0$). The integrated collector is composed of a high permeability material (permeability $K_P$) surrounded with a high thermal conductivity material (thermal conductivity $k_p$). The gas (**G**) enters the SGR from the center of the disc, and diffuses into the reactive porous material (S1) with the help of the integrated collector (IC). The gas and solid materials generate chemical reaction to produce a new solid (S2) as shown in following

$$S1 + G \rightleftarrows S2 \tag{1}$$

Consider a steady state of the SGR, it consumes gas uniformly at the volumetric rate $\dot{m}'''$, and generates heat uniformly at the volumetric rate $q'''$. To couple the heat transfer with mass transfer of the SGR, the relationship between $q'''$ and $\dot{m}'''$ is assumed as $q''' = -\Delta H \cdot \dot{m}'''$ [32,33], where $\Delta H$ is the enthalpy of the reaction. The heat is collected at the center of the disk, but the transfer direction of the gas is opposite to that of the heat. Moreover, the reaction in Eq. (1) can be implemented in the reverse direction.

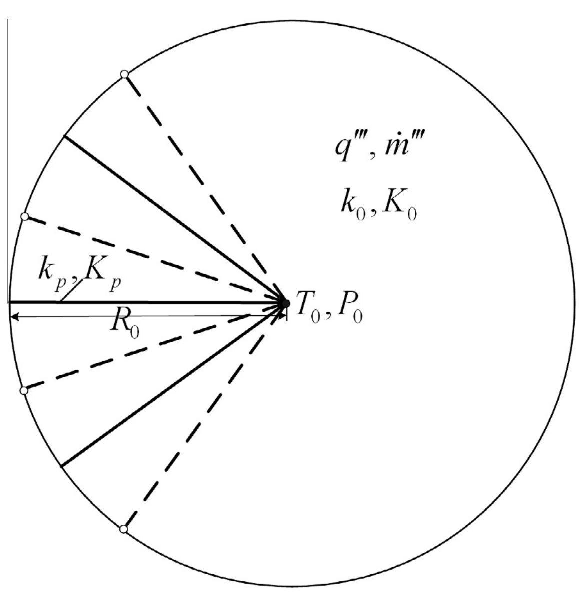


**Fig. 1.** Solid–gas reactor in a radial-pattern disc [37].

As shown in Fig. 2, the radial-pattern disc (RPD) comprises a number ($N$) of equal elemental sectors. The temperature and pressure at the apex of the sector are $T_0$ and $P_0$, respectively. Except for this apex, the boundaries of the elemental sector (ES) are both adiabatic and impermeable ones. When $N >> 1$, the shape of the ES is slender, and it can be treated as an isosceles triangle (base $2H_0$ and height $R_0$). For the specified area ($A_0 = H_0 \times R_0$) and third dimension ($W$) of the ES, its aspect ratio ($H_0/R_0$) can be varied.

When the aspect ratio, the volume fraction ($\phi_0 = D_0/H_0$) of the IC, the thermal conductivity, the permeability and the sector's geometric sizes satisfy the following relationships

$$H_0/R_0 << 1, \phi_0 << 1, k_p >> k_0, K_p >> K_0, W >> H_0, W >> R_0 \tag{2}$$

the heat and mass transfer directions almost go along $y$-axis in the reactive porous material (RPM), and go along $r$-axis in the IC.

For a steady state, the transfer equations in the RPM of the ES are [32,33]

$$k_0 \frac{\partial^2 T}{\partial y^2} + q''' = 0 \tag{3}$$

$$K_0 \frac{\partial^2 P}{\partial y^2} + \dot{m}''' \nu = 0 \tag{4}$$

where $\nu$ is the kinematic viscosity. The corresponding boundary conditions can be given as

$$y = \frac{H_0 - D_0/2}{R_0}(R_0 - r) + \frac{D_0}{2}, \ \frac{\partial T}{\partial y} = 0, \ \frac{\partial P}{\partial y} = 0 \tag{5}$$

$$y = \frac{D_0}{2}, \ T\left(R_0, \frac{D_0}{2}\right) = T_0, \ P\left(R_0, \frac{D_0}{2}\right) = P_0 \tag{6}$$

According to the definition of the EDR, when the EDR caused by gas heat conduction is ignored, the EDR in any element of the RPM can be given by

$$\dot{E}_{h\phi 0,y} = k_0 \left(\frac{\partial T}{\partial y}\right)^2 + \frac{K_0 T}{\mu} \cdot \left(\frac{\partial P}{\partial y}\right)^2 \tag{7}$$

where $\mu$ is dynamic viscosity. From Eq. (7), the EDR in the RPM becomes

$$\begin{aligned} \dot{E}_{vh\phi 0,y} &= 2\int_0^W dz \int_0^{R_0} dr \int_{\frac{D_0}{2}}^{\frac{H_0 - D_0/2}{R_0}(R_0 - r) + \frac{D_0}{2}} \dot{E}_{h\phi 0,y} dy \\ &= R_0 W (2H_0 - D_0)^3 \{210 D_0 K_0 k_p q'''^2 \mu \\ &\quad + \dot{m}'''^2 \nu^2 \{q'''(2H_0 - D_0)[7D_0 \times (2H_0 - D_0)k_p \\ &\quad + 64 k_0 R_0^2] + 210 D_0 k_0 k_p T_0\}\}/(10080 D_0 k_0 K_0 k_p \mu) \end{aligned} \tag{8}$$

In Eqs. (7) and (8), when the pressure tends to be constant in the SGR, the entransy dissipation brought by heat transfer is only considered, therefore, the heat conduction model [23] is the special case of the model of this paper. When the temperature tends to be constant in the SGR, the entransy dissipation brought by mass transfer is only considered, therefore, the mass transfer model is a special case of the model of this paper.

The transfer equations in the IC of the ES are

$$k_p \frac{\partial^2 T}{\partial r^2} + q''' \cdot \frac{(2H_0 - D_0)(R_0 - r)}{D_0 R_0} = 0 \tag{9}$$

$$K_p \frac{\partial^2 P}{\partial r^2} + \dot{m}''' \nu \cdot \frac{(2H_0 - D_0)(R_0 - r)}{D_0 R_0} = 0 \tag{10}$$

The corresponding boundary conditions can be given as

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