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Constructal entropy generation rate minimization for asymmetric vascular networks in a disc-shaped body

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ABSTRACT

Based on constructal theory, the vascular networks with asymmetric pairing in a disc-shaped body are optimized by taking the minimizations of the dimensionless entropy generation rate and dimensionless entropy generation ratio as optimization objectives, respectively. The results show that there exist optimal tube lengths and angles which lead to the minimum dimensionless entropy generation rate and dimensionless entropy generation ratio of the vascular networks with two and three levels of asymmetric pairing, respectively. The optimal constructs of the vascular networks based on asymmetric and symmetric designs are different. For the specified heat flow per unit length on each tube surface, when the number of outlets $N = 24$ and the dimensionless mass flow rate $M_1^* = 10^{-2}$, the dimensionless entropy generation rate with two levels of pairing based on asymmetric design is decreased by 7.80% than that based on symmetric design; when the number of outlets $N = 24$ and the dimensionless pumping power $\tilde{W}_2^* = 1$, the dimensionless entropy generation rate with three levels of pairing based on asymmetric design is decreased by 6.78% than that based on symmetric design. Moreover, the performance improvements of the vascular networks with asymmetric design can also be found for the specified heat flux on each tube surface. The optimization results of the vascular networks based on minimum flow resistance are special cases of those based on minimum entropy generation rate in this paper.

1. Introduction

Constructal theory [1–15] was firstly proposed by Bejan in 1996, and the corresponding constructal law could be stated as [1]: “for a finite-size flow system to persist in time (to live), its configuration must change in time such that it provides easier and easier access to its currents (fluid, energy, species, etc.)”. Various tree-shaped flow structures with multiple scales [16–28] were anticipated by this theory, which showed the advantages of this theory in explaining and guiding the designs both in nature and manmade.

The vasculatures are one of the typical multiple scale structures [16–28], some scholars investigated the thermodynamic performances of vascular networks by considering their thermal and flow performances simultaneously. Zimparov et al. [26] optimized the T-shaped vascular network based on entropy generation rate minimization, and analyzed the entropy generation ratio performance of the H-shaped vascular network in a square area. They further optimized the Y-shaped vascular network in a disc-shaped area, and used the optimal flow structure based on minimum global flow resistance to analyze the thermodynamic performance of the vascular network with several levels of symmetric pairing. These works provided a simple and approximate method to evaluate the performances of the vascular networks. Zimparov et al. [27] further investigated the H-shaped vascular network in a square area and Y-shaped vascular network in a disc-shaped area with constant channel wall temperature, and obtained the thermodynamic performances of the H- and Y-shaped vascular networks different from those in Ref. [26]. Xie et al. [28] optimized line-to-line vascular networks with convective heat transfer by taking entropy generation rate as optimization objective. Based on the model of H-shaped vascular network in Ref. [26], Feng et al. [29] optimized the entropy generation performance of an X-shaped vascular network, and concluded that the performance of the X-shaped vascular network was better than that of the corresponding H-shaped one when the mass flow rate is fixed.

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Nomenclature

c_p	specific heat, J/kg/K
D	diameter, m
h_i	heat transfer coefficient, W/m^2/K
k	thermal conductivity, W/m/K
L	length, m
M	dimensionless mass flow rate
$\dot{m}$	mass flow rate, kg/s
N	number of the outlets
N_s	entropy generation ratio
Nu	Nusselt number
q'	heat flow per unit tube length, W/m
q''	heat flux, W/m^2
R	radius, m
$\dot{S}_{gen}$	entropy generation rate, W/K
T	temperature, K
V	volume, m^3
W	pumping power, W

Greek symbols

β	angle, rad
ν	kinematic viscosity, m^2/s
ρ	density, kg/m^3

Subscripts

in	inlet
m	minimum
opt	optimal

Superscript

$\sim$	dimensionless

The tree-shaped networks discussed above are symmetric ones. Actually, many tree-shaped networks do not present in symmetric forms, but asymmetric ones [30]. Gosselin and Bejan [31] optimized the asymmetric tree flow networks in a disc-shaped body by taking minimum power pumping as optimization objective, and discussed the effects of the asymmetric mass flow rate and main branches on the optimal constructs of the tree flow networks. The results showed that the power pumping requirement of the tree flow networks based on asymmetric design can be reduced effectively than that based on symmetric design. Wechsatol et al. [32] discussed the question about the symmetric or asymmetric designs of the dendritic tree networks. The results showed that the relation between flow resistance and flow fraction at each bifurcation node determined the asymmetric characteristic of the dendritic tree network, and asymmetric bifurcation helped to improve the flow performance of the dendritic tree network. Luo et al. [33] discussed the flow resistance of the asymmetric two-bifurcation network with laminar flow regime, and compared the performances of the asymmetric and symmetric two-bifurcation networks. The results showed that the symmetric two-bifurcation network was not always prefect due to its increment in flow resistance of the network. Wang et al. [34] compared the flow and thermal performances of the three-dimensional symmetric and asymmetric, tree- and leaf-like branching networks. The results showed that asymmetric leaf-like branching network could improve its flow performance obviously when the temperature difference between the inlet and outlet of the fluid maintained its maximum, and the leaf-like networks showed a simpler design and a better thermal performance than those of the tree-like networks with high branching levels.

Based on the models of the tree-shaped fluid networks with asymmetric pairing in Ref. [31] and the Y-shaped vascular networks in Ref. [26], a model of the vascular networks with asymmetric pairing in a disc-shaped body will be considered in this paper. By releasing the symmetric assumption, the optimal constructs of the vascular networks with asymmetric pairing will be obtained based on the minimum dimensionless entropy generation rate, and performance comparisons of the vascular networks with asymmetric and symmetric pairings will be carried out.

2. Vascular network with two levels of asymmetric pairing

A vascular network with one level of symmetric pairing in a disc-shaped body is shown in Fig. 1 [31]. If one tries to release the symmetric assumption, numerical calculation shows that the pairing of the vascular network remains the symmetric one when the optimal performance of the vascular network with one level of pairing is achieved. How about this property when the structure of the vascular network becomes complex? Therefore, one can consider the vascular network with two levels of asymmetric pairing in a disc-shaped body as shown in Fig. 2 [31]. In the disc-shaped body (radius R) of Fig. 2, the fluid (mass flow rate $\dot{m}_T$, temperature T_{in}, density ρ, and thermal conductivity k) enters the inlet of the vasculature from the center of the disc, flows through two levels of tree-shaped round tubes (four kinds of tubes with diameter D_i and length L_i, $i = 01, 02, 1, 2$), and finally flows out of the vascular network from a number (N) of the outlets located the rim of the disc. The flow in each tube is the incompressible viscous and laminar fully developed one. It is assumed that the heat flow per unit tube length q' on each tube surface is constant, and the

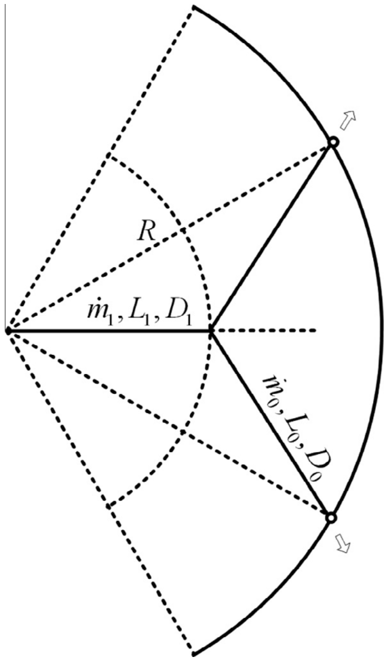

Fig. 1. Vascular network with one level of symmetric pairing in a disc-shaped body [26].