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Estimation of the local convective heat transfer coefficient in pipe flow using a 2D thermal Quadrupole model and Truncated Singular Value Decomposition

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ABSTRACT

The techniques for solving the Inverse Heat Conduction Problem represent useful tools for designing heat transfer apparatuses. One of their most challenging applications derives from the necessity of catching what happens inside a heat transfer apparatus by monitoring the temperature distribution on the external wall of the device, possibly by means of contactless experimental methodologies. The research presented here deals with the application of a solution strategy of the Inverse Heat Conduction Problem (IHCP) aimed at estimating the local heat transfer coefficient on the internal wall surface of a pipe, under a forced convection problem. The solution strategy, formulated for a 2D model, is based on the Quadrupole Method (QM) coupled to the Truncated Singular Value Decomposition approach, used to cope with the ill-conditioning of the problem. QM presents some advantages over the more classical domain or boundary discretization methods as for instance the fact that, being meshless, brings to a reduction of the computational cost. The analytical model, built under the QM, is validated by means of numerical simulations and the numerical outputs are then used as synthetic data inputs to solve the IHCP. The estimation methodology is also applied to experimental data regarding a forced convection problem in coiled pipes. Moreover, the adopted solution technique is compared to other two well-known and consolidated approaches: Finite Element Method coupled to the Tikhonov Regularization Method and Gaussian Filtering Technique. The comparison highlights that, for the problem here investigated, the Quadrupole Method coupled to the Truncated Singular Value Decomposition and Finite Element Method coupled to the Tikhonov Regularization Method perform better than the Gaussian Filtering Technique when the noise level is low, while, for higher noise level values, their efficiency is almost comparable, as it happens in the considered experimental study case.

1. Introduction

A very challenging task in many engineering applications derives from the necessity of catching what happens inside a heat transfer apparatus by monitoring a physical quantity from outside. This could be very useful in cases in which it is not possible to adopt intrusive experimental methodologies to inspect the inside of the apparatus, for mechanical or safety reasons, like for instance in nuclear applications or in any internal flow applications where the internal surfaces cannot be directly surveyed. A straightforward solution for this interesting issue, can be found in the formulation of the Inverse Heat Conduction Problem (IHCP) in the bounding wall of the device, by monitoring, possibly by means of contactless experimental methodologies, the external wall surface temperature distribution.

Among the several possible applications of this approach, the one that aims at estimating the heat flux or the convective heat transfer coefficient distribution over the internal wall of a pipe, starting from the external wall temperature distribution, has proved to be newsworthy within the applied research on heat transfer.

Some successful applications of this methodology have been presented with regards to the forced convective heat transfer in coiled pipes [1]. In these geometries, in fact, the convective heat transfer coefficient varies significantly along the wall periphery, due to the flow pattern that develops as a consequence of curvature and the necessity of monitoring its distribution often then arises in industrial applications [2].

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Nomenclature

Symbol	Quantity (SI Unit)
E_h	global relative estimation error on the heat transfer coefficient h_{int}, Eq. (45)
E_θ	global relative error on the relative reconstructed temperature θ_{ex}, Eq. (32)
$\boldsymbol{F}$	Quadrupole matrix, Eq. (37)
$\boldsymbol{G}$	Quadrupole matrix, Eq. (30)
h	convective heat-transfer coefficient (W/m^2 K)
$\boldsymbol{H}$	Quadrupole external transfer matrix, Eq. (16)
k	wall thermal conductivity (W/m K)
$\boldsymbol{M}$	Quadrupole wall transfer matrix, Eqs. (14) and (24)
N_α	number of measurements
N_h	number of harmonics
q	convective heat flux (W/m^2)
q_g	internal heat generation per unit volume (W/m^3)
r	radial coordinate (m)
Re	Reynolds number
S	surface
t	truncation parameter
T	temperature (K)
$\boldsymbol{R}$	Quadrupole matrix Eq. (35)
R_{env}	overall heat-transfer resistance between the external tube wall and the surrounding environment (m^2 K/W)
y	temperature, Eq. (38) (K)
$\boldsymbol{X},\boldsymbol{Y}$	Quadrupole heat generation terms, Eqs. (26) and (27)
α	angular coordinate (rad)
δ_{n0}	Kronecker's symbol
θ	relative temperature (K)
φ	radial heat flux, Eq. (5) (W/m^2)
Ψ	noisy temperature (K)
Ω	root mean square residual, Eq. (43) (K)

Subscripts, superscripts

b	bulk
env	environment
ext	external
int	internal
$meas$	affected by error
mo	model
T	transpose of a matrix
$\sim$	Fourier transform

Regarding IHCPs, several solution strategies have been suggested under both the parameter estimation and function estimation approach [3] adopting both numerical and analytical models.

Among the numerical solution techniques a relative new and particularly interesting method for the solution of problems in which the boundary is of major importance or requires special attention, is represented by the singular boundary method (SBM) [4,5] which presents some advantages over the more classical domain or boundary discretization methods, as for instance the fact that it is meshless that brings to a reduction of the computational cost.

Analogous advantages are provided also by the Quadrupole Method (QM) which is an explicit analytical method of representation of linear systems available for simple geometries. There are different versions of this technique, depending on the type of problem under study, but there is anyway a common based methodology. In its simpler version in heat conduction its core are 2×2 matrices that link transforms of both temperature and heat flux on one surface of the body under analysis to the same quantities on another surface [6]. The transformation could be both in Fourier and Laplace space. One of its main interests is to get analytical solutions in the transformed domain. This approach will be hereby used in a cylindrical geometry, where a Fourier transformation along the angular coordinate will be implemented.

However, this approach does not completely bypass all the difficulties embedded in the IHCP, in particular it does not solve the complication due to the ill-conditioned character of the problem. In order to bypass this crucial issue, inevitably embedded in the inverse problem formulation, many techniques based on the processing of the experimental data have been suggested and validated in literature.

Among these techniques, the function specification methods [3,7], iterative methods [8–10], methods based on filtering proprieties [11–14] and regularization techniques[15–17] are found.

Regarding the regularization techniques, Tikhonov Regularization Method [16] and Truncated Singular Value Decomposition (TSVD) [17] are two of the most common.

The TSVD method has been applied to solve IHCP in steady (see for instance [18,19]) and transient regimes (see for instance [20]). The stabilizing effect of this method is based on the elimination of the highest modes of the input signal and it has been assimilated to a data filtering approach [21].

The present paper deals with the estimation of the local heat transfer coefficient at the fluid-internal wall interface in forced convection pipe flow problems (see for instance [22] for the same kind of cylindrical geometry); the solution strategy, based on the formulation of the IHCP in the wall solid domain and on the use of the external wall surface temperature distribution as experimental input data, is developed by applying the Quadrupole Method associated to TSVD.

In order to assess the performance of the proposed approach, the results are compared with the ones obtained with two other well known and consolidated techniques: Finite Element Method (FEM) coupled to Tikhonov Regularization Method (TRM) [1] and Gaussian Filtering Technique (GFT) [23].

2. Problem's definition

The objective of this work is to present and validate a procedure to estimate the local convective heat transfer coefficient on the internal side of a pipe's cross section, in a forced convection problem. The problem under test is schematized in Fig. 1 where the cross section of the pipe is shown together with the adopted coordinate system: the fluid flows internally, while the external pipe's wall is exposed to a uniform temperature environment T_{env} and a uniform heat generation q_g is considered within the solid wall. The test section is modeled as a 2D solid domain since the temperature gradient along the tube's axis is assumed to be negligible with respect to the one along the angular direction. The circular section presents an internal radius r_{int}, an external radius r_{ext} and the wall is characterized by a thermal conductivity k.

2.1. Analytical model

Under the conditions above described, the local steady state energy balance equation in the solid domain is expressed as follows: