International Journal of Heat and Mass Transfer 89 (2015) 75–89

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Instability of a horizontal porous layer with local thermal non-equilibrium: Effects of free surface and convective boundary conditions

A. Barletta *, M. Celli, H. Lagziri

Department of Industrial Engineering (DIN), Alma Mater Studiorum Università di Bologna, Viale Risorgimento 2, 40136 Bologna, Italy

ARTICLE INFO

Article history:
Received 27 April 2015
Received in revised form 5 May 2015
Accepted 5 May 2015

Keywords:
Porous medium
Local thermal non-equilibrium
Linear stability
Darcy–Bénard problem
Biot number

ABSTRACT

The onset of convective instability in a horizontal porous layer saturated by a fluid is studied. A regime of local thermal non-equilibrium (LTNE) is considered. The lower plane boundary is an impermeable isothermal wall, while the upper boundary is a free surface with finite heat transfer coefficients to the external environment. The temperature boundary conditions at the free surface are parametrised by two Biot numbers: one for the solid and one for the fluid. A modal linear stability analysis of the basic motionless state is carried out. Limiting cases allowing for an analytical dispersion relation are defined and analysed. The general regime of linear instability is investigated by a numerical solution of the governing differential equations for the normal modes. The neutral stability condition, as well as the critical value of the Darcy–Rayleigh number, is studied in some asymptotic cases and in the general case.

1. Introduction

The Darcy–Bénard problem is a cornerstone for the research area of convection in porous media. It embodies the situation where convection cells are generated in a fluid-saturated porous layer heated from below, with an imposed temperature difference across the layer. The classical setup, modelled by a pair of parallel isothermal walls bounding the porous layer, has been widely studied in the last decades. Analyses of the classical setup, as well as of variants involving isoflux boundaries and/or free boundaries, are presented in review papers and books [1–4].

Beyond the validity of the Oberbeck–Boussinesq approximation and of Darcy's law, the usual assumption made in these studies is that local thermal equilibrium (LTE) between the solid phase and the fluid phase occurs. On the other hand, it is well-known that departures from local thermal equilibrium in a porous medium could arise under highly unsteady conditions, or when sensible differences between the thermal conductivities of the two phases are present [4–6]. Under local thermal non-equilibrium (LTNE), the usual local energy balance equation for the porous medium is actually split into two independent equations involving distinct temperature fields, one for the solid and one for the fluid. A coupling term present in both energy balance equations describes the inter-phase heat transfer through a suitable coefficient, h. When this coefficient of inter-phase heat transfer becomes larger and larger, the asymptotic condition of LTE is approached and the classical analysis of the Darcy–Bénard problem is recovered. A major problem of LTNE is to estimate the coefficient h. Rees [7,8] analysed this problem for various classes of materials.

Banu and Rees [9] reformulated the Darcy–Bénard problem in the LTNE regime and pointed out that the main dimensionless parameters governing the transition to instability are the inter-phase heat transfer parameter, H, and the thermal conductivity ratio γ. The general effect of a departure from LTE is a destabilisation of the horizontal porous layer [9]. The analysis of the effects of LTNE on the Darcy–Bénard instability has been further developed in the last years considering several aspects such as modified boundary conditions, non-Darcy effects, internal heat generation, throughflow, mass diffusion, heterogeneous or anisotropic porous media, and inclination of the layer to the horizontal [10–26].

The aim of this paper is to further develop the present knowledge about the LTNE effects on the Darcy–Bénard instability in a horizontal porous layer. Our analysis will be focussed on a porous layer where the upper boundary is free. In other words, the momentum boundary condition on the upper boundary is one of uniform pressure. The temperature boundary conditions at the upper boundary will be expressed as third-kind (or Robin)

* Corresponding author. Tel.: +39 051 2093287; fax: +39 051 2093296.
E-mail addresses: antonio.barletta@unibo.it (A. Barletta), michele.celli3@unibo.it (M. Celli), hajar.lagziri@studio.unibo.it (H. Lagziri).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.05.026
0017-9310/

Nomenclature

a	dimensionless wave number, Eq. (27)
b	dimensionless exponential coefficient, Eq. (27)
B_s, B_f	(solid, fluid) Biot numbers, Eq. (5)
B_m	effective Biot number, Eq. (18)
c	heat capacity per unit mass [J/(kg K)]
$c_{1\cdots6}$	dimensionless coefficients, Eq. (31)
$\mathbf{c}$	dimensionless vector of the coefficients $c_{1\cdots6}$
$\mathbf{e_z}$	unit vector along the z-axis
g	modulus of the gravitational acceleration, [m/s^2]
$\mathbf{g}$	gravitational acceleration, [m/s^2]
h	inter-phase heat transfer coefficient [W/(m^{3}K)]
H	dimensionless inter-phase heat transfer coefficient, Eq. (5)
i	imaginary unit
k	thermal conductivity [W/(m K)]
K	permeability [m^2]
L	layer thickness [m]
$\mathbf{M}$	dimensionless 6×6 square matrix, Eq. (35)
n	integer
r	dimensionless growth rate, $b = r - i\omega$
R	Darcy–Rayleigh number, Eq. (5)
$\widetilde{R}_1, \widetilde{R}_2, \widetilde{R}_3$	rescaled Darcy–Rayleigh numbers, Eqs. (34), (43) and (46)
$\mathcal{R}$	modified Darcy–Rayleigh number, Eq. (47)
$\Re$	real part
s_1, s_2	dimensionless parameters, Eq. (49)
t	dimensionless time, Eq. (3)
T	dimensionless temperature, Eq. (3)
T_m	effective local dimensionless temperature, Eq. (14)
T_w	wall temperature [K]
T_0	reference temperature [K]
$\mathbf{v}$	dimensionless velocity field (u, v, w), Eq. (3)
$\mathbf{x}$	dimensionless Cartesian coordinates (x, y, z), Eq. (3)

Greek symbols

α	thermal diffusivity [m^2/s]
β	thermal expansion coefficient [K^{-1}]
γ	thermal conductivity ratio, Eq. (5)
ΔT	reference temperature difference, $T_w - T_0$, [K]
ϵ	dimensionless perturbation parameter, Eq. (21)
$\zeta_{1\cdots3}$	roots of Eq. (33)
Θ	dimensionless temperature perturbation of the fluid phase, Eq. (8)
λ	dimensionless parameter, Eq. (5)
Λ	average dimensionless temperature, Eq. (30)
μ	dynamic viscosity [Pa s]
ν	kinematic viscosity [m^2/s]
$\xi_{1\cdots6}$	dimensionless coefficients, Eq. (32)
ρ	density [kg/m^3]
φ	porosity
$\hat{\Psi}$	dimensionless streamfunction, Eq. (24)
$\tilde{\psi}$	rescaled streamfunction amplitude, Eq. (39)
ψ, θ_s, θ_f	dimensionless amplitudes of the normal modes, Eq. (23)
ω	dimensionless frequency, $b = r - i\omega$

Subscripts, Superscripts

$*$	dimensional quantity
—	complex conjugate
$\hat{}$	perturbation of the basic state, Eq. (21)
b	basic state
c	critical value
f, s	fluid phase, solid phase

boundary conditions by adopting, in general, different Biot numbers for the solid and the fluid.

To the best of our knowledge, in the literature there exists only one study of LTNE convective instability in the presence of free boundaries, namely that carried out by Postelnicu and Rees [10]. However, these authors developed their investigation by adopting a Darcy–Brinkman–Forchheimer model where the appropriate free boundary conditions are actually stress-free conditions, instead of uniform pressure conditions. The consequence is that the analysis developed by Postelnicu and Rees [10] does not yield the stability analysis with free boundary conditions (uniform pressure) in the limit of Darcy's regime. On the other hand, in the limiting case where Darcy's law holds, the analysis presented by Postelnicu and Rees [10] reduces to that of Banu and Rees [9] for impermeable isothermal boundaries.

2. Problem statement and governing equations

We consider a plane porous layer with uniform thickness L. A uniform temperature, T_w, is prescribed at the lower impermeable wall, $z^* = 0$. The upper boundary is a free surface, with a uniform pressure, which exchanges heat with an external fluid environment having a reference temperature, T_0, such that $T_0 < T_w$ (see Fig. 1). The fluid saturated porous medium is studied according to the following assumptions:

- Darcy's law holds;
- the porous medium is homogeneous and isotropic;
- the Oberbeck–Boussinesq approximation [3,4] can be applied;
- viscous dissipation is negligible;
- the hypothesis of local thermal equilibrium (LTE) does not hold.

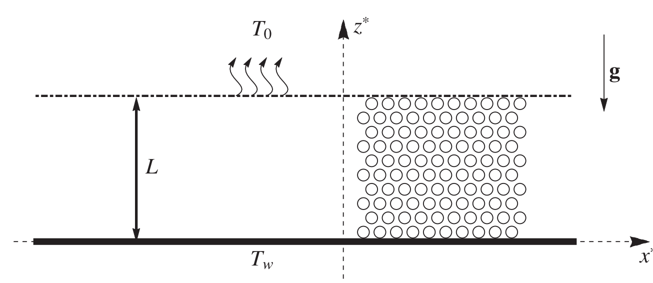

Fig. 1. Sketch of the porous layer.

The local thermal non-equilibrium (LTNE) model is based on the definition of two different local temperatures, one for the solid and one for the fluid. The inter-phase heat transfer rate is modelled through a constant coefficient, h, multiplying the local temperature difference between the phases. Thus, two energy balance equations have to be written for the two phases. The governing equations can be written as [4–6]

$$\nabla^* \cdot \mathbf{v}^* = 0, \tag{1a}$$

$$\frac{\mu}{K} \nabla^* \times \mathbf{v}^* = \rho_f g \beta \nabla^* \times \left[(T_f^* - T_0) \mathbf{e}_z \right], \tag{1b}$$

$$(1 - \varphi) \frac{\partial T_s^*}{\partial t^*} = (1 - \varphi) \alpha_s \nabla^{*2} T_s^* + \frac{h}{(\rho c)_s} \left(T_f^* - T_s^* \right), \tag{1c}$$

$$\varphi \frac{\partial T_f^*}{\partial t^*} + \mathbf{v}^* \cdot \nabla^* T_f^* = \varphi \alpha_f \nabla^{*2} T_f^* - \frac{h}{(\rho c)_f} \left(T_f^* - T_s^* \right). \tag{1d}$$

Eq. (1b) is the vorticity representation of Darcy's law, while f, s denote the properties of the fluid and of the solid, respectively. Asterisks indicate dimensional variables and operators.