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Falling liquid films on a slippery substrate with Marangoni effects

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ABSTRACT

This paper investigates the dynamics of liquid films of mean thickness h_0 flowing down a uniformly heated slippery substrate. The dimensionless slippery length β is small and is assumed to have the same order as the film parameter $\epsilon = h_0/l$ where l is a typical wave length. A weighted-residual model and a Benney-type model are derived to discuss the influences of slippery length β, Reynolds number Re, Marangoni number M and Biot number Bi on the dynamics of the film. It is found that, when Reynolds number is small ($Re < 1$) and the wall is non-slip, the Benney-type model, weighted-residual model are in good agreements with the linearized Navier–Stokes equations (LNS). However, when the wall is slip or the Reynolds number is moderate, the Benney-type model does not agree with LNS. Our results show that, for a small slippery effect at the substrate, the weighted-residual model compares well with LNS at moderate Re. It demonstrates that the weighted-residual model is more reasonable than the Benney-type model. A critical Reynolds number obtained by the weighted-residual model for this system is $Re_c = \frac{35(1-3\beta)^4}{6(7-67\beta+159\beta^2)}\left(Wek^2 + \cot\alpha - \frac{3MBi}{2(1+Bi)^2}\right)$ (α is the inclined angle and We is the Weber number). Linear stability analysis shows that, the instability is enhanced as the slippery length β and the Reynolds number Re increase. In the long-wave range, the linear wave speed $c = 1 + 2\beta$. When the wall is heated, the instability is reinforced. The instability is impeded when the substrate is cooled. Nonlinear analysis by the traveling wave solutions shows that, the wave speed and wave height of traveling waves are promoted by the slippery and fluid inertial effects. Nonlinear study shows that the traveling wave speed is promoted by a heated substrate and is reduced by a cooled substrate. It shows that the speed of fast waves is larger than the linear wave speed and the higher the wave is the faster it travels. Direct numerical simulation of the weighted residual model supports the linear stability analysis and the nonlinear analysis of traveling wave solutions.

1. Introduction

A gravity-driven liquid film on an inclined plane is an open flow system and a large number of theoretical and experimental studies have been devoted to this field [1]. The modeling work of such a flow system was pioneered by Benney [2]. Based on the evidence that the wave length of a typical wave l is much longer than the film mean thickness h_0, Benney introduced a small film parameter $\epsilon = h_0/l$ and reduced the full governing equations asymptotically [2]. A single equation governing the thickness of the liquid interface (h) was then derived wherein all the physical parameters (velocity and pressure) were slaved to h. Thanks to the work of Benney, many works have been carried out by extending Benney's work. For instance, Joo et al. investigated a heated film wherein the evaporating effect was taken into account [3]; Scheid et al. considered the effect of non-uniform heating on the linear and nonlinear dynamics of a falling film by deriving a Benney-type equation [4]; Thiele et al. considered a thin liquid film falling down a porous heated substrate [5]. However, Pumir et al. indicated that the Benney equation blows up without a bound when the Reynolds number is moderate which gives non-physical solution [6]. This failure is related to the strict slaving of the velocity field to the film thickness h.

To address the blow-up of Benney equation, Ooshida regularized the Benney equation by the Padé approximation [7]. However, the regularized equation is in poor agreement with experiments and the data by direct numerical simulation of the full Navier–Stokes equations even though the singularity phenomenon is removed [7]. Shkadov [8] proposed an integral boundary layer model which introduced one more degree of freedom. The integral boundary layer model solves two coupled equations governing the film thickness h and the local flow rate q. This method was

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extended by Kalliadasis to investigate the dynamics of a heated falling film wherein the Marangoni effect on the interfacial instability was discussed [9]. The integral boundary layer model has been extended to study the problem of thin liquid films flowing down vertical cylinders [10,11]. Nevertheless, Trihonov [12] found that the integral boundary layer model differs from the full Navier–Stokes equations starting from some values of the Kapitza number. Another problem encountered is that, when the inclined angle α is moderate, $0 < \alpha < \pi/2$, the integral boundary layer model predicts wrong linear stability threshold. Furthermore, asymptotic expansion of the integral boundary layer model does not agree with the Benney equation even if the Reynolds number is small. The inaccuracy of the integral boundary layer model is due to the assumption of the parabolic velocity profile which causes the error in the prediction of the shear stress at the plate.

Motivated by the inconsistency between the integral boundary layer model and the Benney equation, Ruyer-Quil and Manneville developed a weighted residual model [13,14] which addresses the problem successfully. Note that, the weight function chosen by Ruyer-Quil and Manneville [14] was proportional to the velocity profile whereas the weight function for the integral boundary layer model can be considered as a constant number 1. Ruyer-Quil et al. further extended the weighted residual model and revisited the problem of a thin liquid film down a vertical cylinder [15]. We observed that, for the linear stability analysis, there is no qualitatively difference between the integral boundary layer model and the weighted-residual model when they are applied to the problem of thin liquid films on vertical cylinders. This is so because the film is unstable due to the Rayleigh-Plateau mechanism rather than the fluid inertia. Scheid et al. extended the weighted residual to a heated liquid film flowing down an incline in which the heat convection was neglected [16]. The comparison between the Benney equation and the weighted residual and the valid domain of the Benney equation were discussed by them [16]. Ruyer-Quil et al. [17] further took into account the effect of heat convection and derived three coupled equations governing the film thickness h, the flow rate q and the interface temperature. Trevelyan et al. examined the influence of a thicker substrate which was cooled by the ambient gases in the other side on the dynamical behavior of thin liquid films [18]. In the study by Trevelyan et al. [18], the substrate is not isothermal. For the modeling work of thin liquid films, we refer the readers to the monograph by Kalliadasis et al. [19] for more information.

In many practical cases, the solid substrate is slip. The surface can be rough at the microscale and the superhydrophobic effect would cause a considerable slippery length. The slippery length can be up to 50 μm in the case of grooved surfaces [20]. The Navier slip boundary condition is therefore applied at the slippery bound. In fact, some previous studies of thin liquid films flowing down a saturated porous substrate reduced the system to a one-sided model, which neglected the hydrodynamics in the porous phase and modeled the problem by a solid impermeable substrate with a Navier-slip boundary condition [21]. The slippery length in such a system [21] is very large. Recently, Ogden et al. derived a weighted-residual model and investigated a falling film on a wavy porous heated substrate where the influence of the porous medium was reduced to the Navier-slip condition [22]. Samanta et al. discussed the instability of an isothermal film falling down a slippery plane by deriving a weighted-residual model [23]. We observed that, the equations derived by Ogden et al. [22] do not agree with the equations by Samanta et al. [23] even if the substrate is flat and the thermal field is removed in the work of Ogden et al. [22]. The inconsistency between the two models is due to the different consideration of the order of the slippery length. In the work of Ogden et al., terms up to second order of the slippery length were retained while higher order terms were neglected [22]. In Samanta et al.'s work [23], all the terms were retained. A careful look at the literature indicates that studies on instability of liquid film flowing down a slippery heated substrate at a moderate flow rate are very limited. In addition, the work by Ogden et al. [22] focused on a wavy porous substrate whereas the nonlinear dynamical behavior of the film on a flat slippery plane was not discussed. This paper investigates a falling liquid film on an inclined flat slippery heated/cooled substrate. A first order weighted residual model is derived to study the problem and the dimensionless slippery length (slippery length/mean film thickness) is assumed to have the same order as the film parameter ϵ.

The rest of the paper is organized as follows. Mathematical formulation is constructed in Section 2. Scalings and the boundary layer approximation are carried out in Section 3. Section 4 presents the derivation of the weighted-residual model. Linear stability analysis is discussed in Section 5. In Section 6, the traveling wave solutions are sought so as to discuss the influences of wall slippery, fluid inertia and thermocapillary force on the wave speed and wave height. The nonlinear simulation of the weighted residual model is performed in Section 7. A conclusion is made in Section 8.

2. Mathematical formulation

We consider a liquid film falling down a uniformly heated/cooled inclined slippery substrate as shown in Fig. 1. The liquid is Newtonian with constant density ρ, kinematic viscosity ν and dynamic viscosity $\mu = \rho\nu$. In this paper, the two-dimensional hydrodynamic problem is considered. Motion of the liquids is governed by the continuity equation and momentum equation,

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g}, \tag{2}$$

where $\nabla = (\partial_x, \partial_y)$ and $\mathbf{u} = (u, v)$. $\mathbf{g}$ is the gravity acceleration and p denotes the pressure in the liquid phase.

Temperature T within the liquid phase is governed by the Fourier equation,

$$\frac{DT}{Dt} = k_{th} \nabla^2 T, \tag{3}$$

where k_{th} is the thermal diffusivity of the liquid.

On the surface of the substrate $y = 0$, we apply the Navier-slippery condition,

$$u = \beta_s u_y, \quad v = 0. \tag{4}$$

Here, β_s is the slippery length. $\beta_s = 0$ corresponds to a non-slip substrate.

At $y = 0$, the temperature boundary condition is specified as,

$$T = T_s. \tag{5}$$

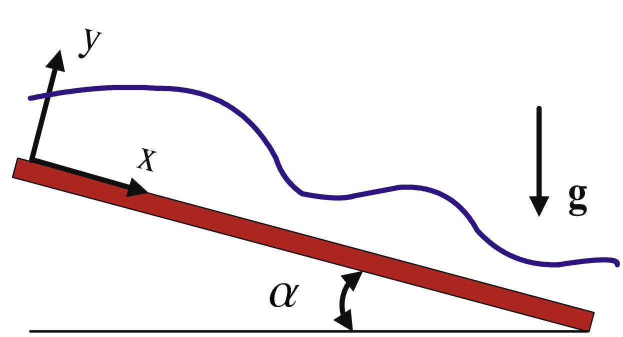

Fig. 1. Geometry of the system. α is the inclined angle.