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Direct and inverse heat transfer in non-contacting face seals

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ABSTRACT

The subject of this paper was to develop a mathematical model of a non-contacting face seal describing the phenomenon of the heat transfer in the system: sealing rings – fluid film. The function of non-contacting face seals used in rotor machines is to separate the working agent from the external environment. The nature of operation of non-contacting face seals allows the fluid to leak through the clearance not bigger than a few micrometres. During the operation of the rotor machine between the co-operating rings, an intense conversion of mechanical energy into heat occurs. At first, the heat flux generated in the fluid film is channelled to the sealing rings and then to the surrounding fluid.

The solution of the presented model was conducted with the use of analytical methods for the direct and the inverse heat transfer problem. The distribution of the temperature fields in the sealing rings for the direct heat transfer problem was determined with the use of Fourier–Bessel series as the surface function of two variables (r, θ) for the cross-section of a ring. The inverse heat transfer problem was solved with the use of Trefftz functions.

The presented computational methods allow a more detailed identification of the phenomenon of the heat transfer in non-contacting face seals and indicate a direction of further research and preparation of new computational methods.

1. Introduction

Determining the temperature distribution in the lubricating fluid and in the structural elements (seal rings) is of great practical importance due to the limitation of failures caused by significant increase of temperature and evaporation of agent as well as the thermo-elastic deformities which are the consequence of the occurrence of large temperature gradients. Formulating models which allow possibly the most precise reproduction of the occurring physical phenomena, i.e. heat transfer, thermal deformations and changes of physiochemical features of the fluid passing through the radial clearance is extremely significant already at the stage of designing the devices in which non-contacting face seals are applied.

The issue of the heat transfer in non-contacting mechanical seals was the subject of a number of academic papers whose results have been published in recent years. For instance, Dumbrava and Morariu [1] presented the thermohydrodynamic (THD) analysis for the mechanical face seal in which they considered the changes of the physical features of the agent and the heat conductivity to the rings limiting the clearance, as well as the heat transfer as convection with the fluid surrounding the rings. Lebeck [2] in his book summarized various heat transfer mechanisms in mechanical seals. Pascovici and Etsion [3,4] presented the method of determining the heat flow in the area of sealing rings and the THD analysis for a double seal in the "face to face" configuration. As a result of an analytical solution, they obtained a radial temperature distribution in the fluid film and in the rotating ring (rotor) assuming that the surface of the second ring (stator) is insulated and all the heat generated in the film is channelled through the rotor to the surrounding fluid. Subsequent thermohydrodynamic (THD)} [5–7] and thermoelastohydrodynamic (TEHD) [8] models of a non-contacting face seal can be found for instance in the paper of Tournerie et al. [9] where basic equations of THD lubrication and the equations of heat conductivity were presented.

In this paper the problem of the heat transfer in a non-contacting face seal was solved with the use of two methods. The first analytical method consisted in separating the variables in a partial differential equation which described the heat transfer in a cylindrical system. As a consequence, ordinary differential

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Nomenclature

C_1, C_2, C_3, C_4	constants
c_n^s	unknown coefficient of the linear combination
h	clearance geometry in the direction of the radial coordinate r
h_o	nominal clearance height
$h(r)$	function of clearance height
$J_0(sr), Y_0(sr)$	Bessel functions of the first and second kinds, respectively
n	direction normal to the surface
r_i, r_o	inner, outer radius, respectively
s_n	constant $\left(\frac{1}{m}\right)$
T	absolute temperature
T^f	fluid temperature in the (r,z) coordinates
T_m	average temperature of the fluid in the clearance
T_o	temperature of the surrounding fluid (generally assumed to be a constant)
V_n	Trefftz functions satisfying Laplace equation
Greek	
α	convection coefficient
β	taper angle
λ^f	fluid thermal conductivity
$\lambda^i - \lambda^s, \lambda^r$	thermal conductivity for the stator and the rotor, respectively
μ_o	dynamic viscosity at T_o
v_ϕ	distribution of the flow velocity of the fluid in the clearance
ω	angular velocity
$\theta(r,z) = T - T_o$	change in the temperature in the (r,z) coordinates
$\theta^i - \theta^s, \theta^r$	changes in the temperature of the stator and the rotor, respectively

equations were obtained whose solutions are presented in the form of Bessel series. Such results may be obtained if relevant boundary conditions are known. In the case when the equation governing the process and the boundary conditions are known it is a direct problem. The inverse problem may be classified into the following categories:

- a boundary inverse problem,
- a coefficient problem,
- an identification of sources,
- a geometric inverse problem,
- an identification of an initial condition.

In the paper the boundary inverse heat conduction problem was solved. In this problem one of the boundary conditions is not known. Such a situation occurs for example when on one boundary of the area, the temperature measurement is not possible and thus, it is impossible to determine the heat flux. That is why in this paper another method of specifying the temperature of the field was suggested which allows both direct and inverse heat transfer to be solved. The method is based on Trefftz functions for the differential equation under consideration. The method is presently quite well recognised for a wide range of linear partial differential equations in various systems of coordinates. For a given differential equation, a complete set of functions, which strictly fulfil that equation (Trefftz functions) is determined and the solution is approximated with the linear combination of Trefftz functions. The coefficients of the linear combination are determined in such a way so that the boundary conditions would be fulfilled in the best way (usually in the method of least squares). Trefftz method was first described in the paper of [10]. Then, many authors developed that method. Herrera, Jirousek, Kupradze, Leon, Sabina, Zieliński and Zienkiewicz should be mentioned herein [11–15]. In the papers mentioned above, stationary problems or problems brought to be stationary through time discretisation are considered. Rosenbloom and Widder in the paper of [16] obtained Trefftz functions for 1D non-stationary heat conduction equation, in which 'time' occurred as one of the variables. A number of papers have been published in this trend. In the papers of [17–21] Trefftz functions were applied to the solutions of problems of linear direct and inverse heat conductivity in various systems of coordinates. The nonlinear issue of heat conductivity was solved in the paper of [22]. Trefftz functions for the wave equation were presented in the papers of [23–26]. Direct and inverse problems of theroelasticity were solved with the use of Trefftz functions in the papers of [27–29]. Trefftz functions related to the problem of beam vibrations were described in [30,31], while to plate vibrations in the papers of [31,32]. Several monographs were written on Trefftz functions which among others include [20,33–35]. Experiences of authors of the above mentioned papers show high effectiveness of Trefftz method in solving the boundary inverse problems described by differential equations.

The use of Trefftz functions in this paper and the results obtained with that method allow to determine the temperature distributions in the sealing rings. The results have been compared with the results obtained on the basis of the solution of the direct heat transfer under specified geometrical and operating parameters of a non-contacting face seal (type FMR).

2. Seal model

The main structural concept related to non-contacting face seals is to maintain a clearance separating the co-operating rings at the level of a few micrometres during operation. The nominal height of the clearance h_i results directly from the balance of forces affecting the sealing rings, mainly the pressure force coming from the elastic element and the hydrostatic force which depends on the distribution of the pressure generated in the clearance between the parallel rings. An overall scheme of a non-contacting face seal was shown in Fig. 1. This type of seal consists (as already mentioned) of two cooperating rings – one of them, the stator (1) which is rigidly mounted in the housing, while the rotor (2) rotates together with the shaft (6) of the rotor machine and is pressed to the stator with a spring (3).

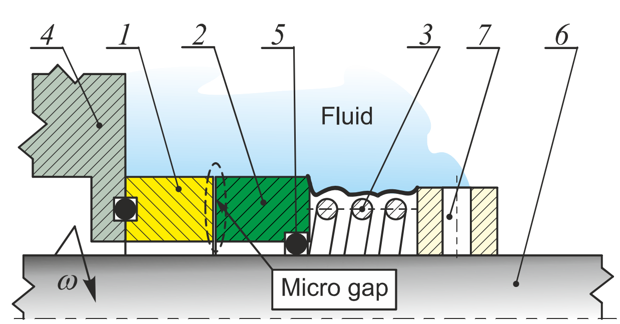

Fig. 1. The scheme of a non-contacting seal. 1 – stator, 2 – rotor, 3 – spring, 4 – housing, 5 – O-ring, 6 – shaft, 7 – locator.