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Conductive and convective heat transfer in fluid flows between differentially heated and rotating cylinders

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ABSTRACT

The flow of fluid confined between a heated rotating cylinder and a cooled stationary cylinder is a canonical experiment for the study of heat transfer in engineering. The theoretical treatment of this system is greatly simplified if the cylinders are assumed to be of infinite length or periodic in the axial direction. In these cases heat transfer in the laminar regime occurs only through conduction as in a solid. We here investigate numerically heat transfer and the onset of turbulence in such flows by using both periodic and no-slip boundary conditions in the axial direction. The influence of the geometric parameters is comprehensively studied by varying the radius ratio ($0.1 \leqslant \eta \leqslant 0.99$) and the length-to-gap aspect ratio ($5 \leqslant \Gamma \leqslant 80$). Similarly, a wide range of Prandtl, Rayleigh, and Reynolds numbers is explored ($0.01 \leqslant \sigma \leqslant 100$, $Ra \leqslant 30{,}000$, and $Re \leqslant 1000$, respectively). We obtain a simple criterion, $Ra \lesssim a(\eta)\Gamma$, which determines whether the infinite-cylinder assumption can be employed. The coefficient a is well approximated by a cubic fit over the whole η-range. Noteworthy the criterion is independent of the Prandtl number and appears robust with respect to Reynolds number even beyond the laminar regime.
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1. Introduction

Instabilities driven by the combination of rotation and thermal gradients determine the dynamics of complex geophysical, astrophysical and industrial flows. Simple models of such flows can be tested in laboratory experiments of a laterally heated differentially rotating annulus, for which extensive information about the physical mechanisms and flow regimes can be found in the literature (see [1] for a review on this topic). The case of rotating heated inner cylinder and stationary cooled outer cylinder (RHISCO) is a model for the cooling of rotating machinery, the solidification of pure metals, techniques of chemical vapour deposition, rotating-tube heat exchangers and nuclear reactor fuel rods [2–5]. The geometry of such an experimental apparatus is fully specified by the length-to-gap aspect ratio $\Gamma = h/(r_o - r_i)$, and the radius-ratio $\eta = r_i/r_o$, where r_i and r_o are the radii of the inner and outer cylinders, and h is their height.

Depending on the geometry, RHISCO experiments used in the literature can be classified in two groups. The first group of experiments [6–8] is characterised by long cylinders $\Gamma = h/(r_o - r_i) \geqslant 100$ and narrow gap $\eta = r_i/r_o \lesssim 1$. Ali and Weidman [9] performed a detailed linear stability analysis of such flows using axial periodicity and reported on the influence of the Prandtl number (σ) and η on the stability boundaries. Their results showed a good agreement with [6] and, to a lesser extent with [7]. Ali and Weidman attributed the discrepancies to the limitations of linear stability theory and the infinite-cylinder idealisation to capture the experimental details. A similar linear stability analysis [10] reported good agreement between numerical and related experimental results [8]. Nonlinear simulations for small temperature gradients were provided by Kedia et al. [11] who quantified the heat transfer across the system. A second group of experiments embraces apparatuses with moderate aspect ratio and wide gap. Ball and Farouk [12–14] reported heat transfer measurements as well as the sequence of flow transitions using an experimental setup with $\Gamma = 31.5$ and $\eta \sim 0.5$. Subsequent numerical simulations [15] for $\Gamma = 10$ and $\eta = 0.5$ provided insight on the bifurcations structure of the system. However, the results showed significant discrepancies with experiments suggesting strong effects of the axial boundaries.

An accurate numerical simulation of the axial (Ekman) boundary layers in flows between long cylinders entails a substantial computational cost, especially for rapid rotation and large temperature gradients. The assumption of axial periodicity reduces the computational effort because the Ekman layers are not present and only a short central fraction of the apparatus needs to be

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simulated to estimate transport properties. Moreover, under this assumption variables can be expanded as a Fourier series in the axial direction, which greatly simplifies the numerical approach and enables the use of more efficient solvers.

In this paper we determine under what conditions periodic boundary conditions can be employed to describe the dynamics of RHISCO experiments. In particular, we provide criteria to distinguish flow features that arise from the interplay between differential rotation and temperature, from those which are mainly determined by the axial boundaries or end walls. We compare the flow dynamics by using both physical (no-slip) and axially periodic boundary conditions in our numerical simulations. We show that axial periodicity renders a good approximation of laboratory flows as long as the Rayleigh number Ra is small. We provide a simple criterion that determines whether heat transfer in the laminar flow is conductive or convective. In particular, conductive profiles, which enable the use of axially periodic boundary conditions, are obtained as long as $Ra < a(\eta)\Gamma + b(\eta)$. In addition, the coefficient b may be neglected for $\Gamma \gtrsim 15$.

2. Specification of the system and numerical methods

We consider the motion of an incompressible fluid of kinematic viscosity ν and thermal diffusivity κ confined in the annular gap between two rigid and concentric rotating cylinders of radii r_i and r_o. The inner cylinder rotates at constant angular velocity Ω, whereas the outer cylinder is kept at rest. A radial thermal gradient is considered by setting the inner and outer cylinder temperature to $T_i = \overline{T} + \Delta T/2$ and $T_o = \overline{T} - \Delta T/2$, respectively. Here $\overline{T}$ is the mean temperature of the fluid. The axis of the cylinders is vertical, i.e. parallel to the gravitational acceleration g. We study flows with stationary end walls and with axially periodic boundary conditions (Fig. 1(a) and (b) respectively). The latter model the case of infinitely long cylinders, whereas the former reproduce experimental boundary conditions (no-slip for the velocity and thermally insulating end walls for the temperature).

2.1. Governing equations

We consider the Boussinesq approximation including centrifugal buoyancy in an inertial reference frame as described in [16]. The dimensionless governing equations are

$$(\partial_t + \mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \nabla^2\mathbf{v} + GT\hat{\mathbf{z}} + \epsilon T\mathbf{v}\cdot\nabla\mathbf{v}, \tag{1a}$$

$$(\partial_t + \mathbf{v}\cdot\nabla)T = \sigma^{-1}\nabla^2 T, \tag{1b}$$

$$\nabla\cdot\mathbf{v} = 0, \tag{1c}$$

where $\mathbf{v} = (u, v, w)$ denotes the velocity field vector and T is the deviation of the temperature with respect to $\overline{T}$. The length, time, temperature and pressure scales chosen to make the set of equations dimensionless are the gap width $d = r_o - r_i$, the viscous time d^2/ν, the temperature difference between the cylinders ΔT and $(\nu/d)^2$ respectively. There are six independent dimensionless numbers (see Table 1). The term $\epsilon T\mathbf{v}\cdot\nabla\mathbf{v}$ accounts for centrifugal buoyancy, including secondary effects stemming from differential rotation or strong internal vorticity [16]. The equations are solved in cylindrical coordinates (r, θ, z)

2.2. Numerical methods

In the axially periodic case the onset of instabilities was determined via linear stability analysis of the basic flow as in [16]. Fully nonlinear simulations were performed using the Boussinesq-approximation [16], which was added with the heat equation to the finite-difference–Fourier–Galerkin (hybrid MPI-OpenMP) code of Shi et al [17]. A time-step $\delta t = 2\times10^{-5}$ viscous time units was used in all computations.

For rigid end walls the governing equations were solved using a second-order time-splitting method. A pseudo-spectral formulation is used for the spatial discretisation, with the Fourier–Galerkin method in the azimuthal coordinate θ and Chebyshev collocation in r and z. The code is based on a previous hydrodynamic code [18], which has been extended with the Boussinesq-approximation of [16] and parallelised as in [17]. Details can be found in Lopez's PhD thesis [19]. The numerical resolution has been chosen to ensure that the infinite norm of the spectral coefficients decays at least four orders in magnitude. Time steps as small as $\delta t = 1\times10^{-5}$ viscous time units have been required for numerical stability and accuracy of the second-order temporal scheme.

3. Conductive and convective basic flows

The assumption of axial periodicity allows to considerably simplify the calculation of the basic flow. The radial velocity is zero

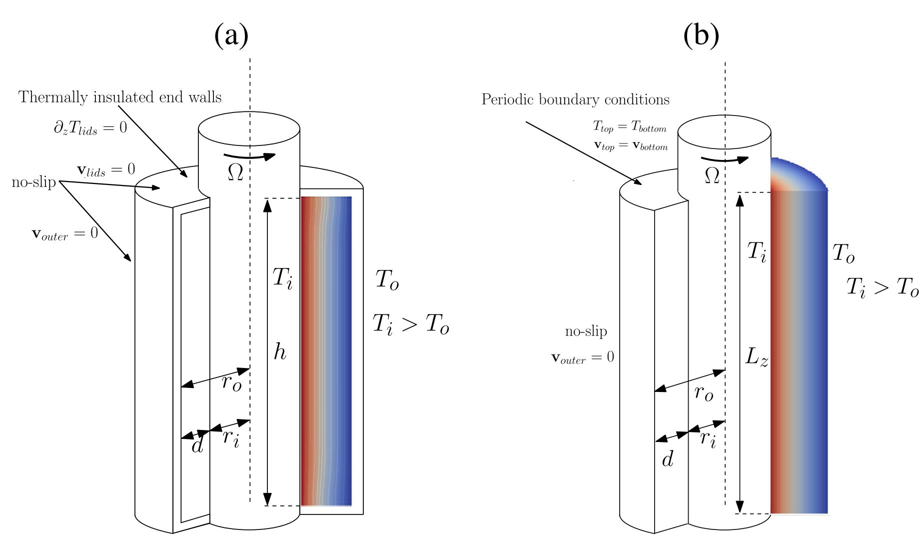

Fig. 1. Sketches of geometry and boundary conditions for the two cases considered in this paper. (a) Stationary insulating endwalls (no-slip boundary condition for the velocity and zero flux for the temperature) and (b) axially periodic boundary conditions. The temperature profile is superimposed on the right hand side of each figure.

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