International Journal of Heat and Mass Transfer 89 (2015) 414–424

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Axisymmetric modeling of the thermal cooling, including radiation, of a circular glass disk

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ARTICLE INFO

Article history:
Received 12 November 2014
Received in revised form 23 April 2015
Accepted 25 April 2015
Available online 5 June 2015

Keywords:
Glass
Tempering
Modeling
Radiation
Temperatures
Residual stresses

ABSTRACT

Achieving correct tempering in thin glass is very important to prevent undesired stress and breakage. Computer simulation can elucidate and control the tempering process. For semitransparent materials like glass, heat transfer by thermal radiation is substantial; for thick glass, it may dominate over convection and conduction. The present paper investigates the tempering of thin glass. A circular glass disk supported by a metallic mold cools down by natural convection. The process can be modeled mathematically by coupling the heat and radiative transfer equation in the glass disk with the heat transfer in the support mold. Even at the glass and support mold interface, radiation exchange must be considered. Mechanical behavior is modeled using the mechanical equilibrium and applying the constitutive law for glass during cooling. For the numerical radiation simulation, the Abaqus® commercial software package was combined with an in-house C code. Based on the differences in temperatures and stresses between simulations that only take surface radiation into account and those that consider surface as well as internal radiation, it has been shown that, even for thin glass, internal radiation cannot be ignored.

1. Introduction

Proper glass cooling is very important to achieve desired product quality. If cooling does not occur properly, undesired stresses may occur inside the glass, which could lead to glass breakage either during the cooling itself or during subsequent product manipulations. Numerical simulations can be used to study the physical behavior of glass (temperature, stresses) during cooling to improve cooling process design. Modeling glass cooling is a complex, non-linear thermo-mechanical problem. In the last decades, glass-cooling models have been widely developed [1–4] using commercial software packages or homemade codes.

Glass is a semi-transparent material. Consequently, in addition to heat conduction and heat convection, radiation plays an important role in thermal exchanges, especially at high temperatures where it is the dominant heat transfer process. Thorough assessments of radiative heat transfer can be found in [5–7], while the application of radiative heat transfer to the glass industry is assessed in [8,9].

In the literature, different solutions were proposed to model glass cooling and account for radiation effects. The simplest one is to completely ignore radiation [4,10]. Another solution is to consider surface radiation only by applying Stefan–Boltzmann's law [11], which is appropriate for opaque materials like metals. However, since glass is a semi-transparent material, internal radiation cannot be ignored. A widely used solution involves treating radiation as a correction of heat conduction by using an equivalent conductivity (such as the active thermal conductivity method [12,13]) or the Rosseland approximation [14]. These methods are fast and simple to integrate into commercial software packages. Originally derived by Rosseland in 1924 for stellar radiation [15], the Rosseland approximation is only valid, however, for optically thick glass, i.e. $d \cdot \kappa(\lambda) \gg 1$, where $\kappa(\lambda)$ denotes the wavelength depending absorption coefficient and d the distance to the boundary. In [3,16] it was shown that the use of the Rosseland approximation for glass cooling could lead to significant stress calculation error.

The right way to model thermal radiation is to use the radiative transfer equation (RTE):

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.04.091
0017-9310/

$$\overline{\Omega}\cdot\overline{\nabla}I(\bar{x},\overline{\Omega},\lambda,t)+\kappa(\lambda)I(\bar{x},\overline{\Omega},\lambda,t)=\kappa(\lambda)B(T(\bar{x},t),\lambda).$$

This equation is non-linear and high-dimensional regarding the spectral radiative intensity $I(\bar{x},\overline{\Omega},\lambda,t)$, and therefore very time consuming to solve. Lee and Viskanta [17] used the Discrete Ordinate Method (DOM) in axisymmetric cylindrical coordinates to solve this equation in the case of the cooling of an optical-quality glass disk by natural cooling. These researchers suggested surrounding the disk with air at a constant temperature. To validate the solution method, they modified the boundary condition to obtain a one-dimensional solution from the two-dimensional formulation and compared it with experimental data from Field and Viskanta [18]. The difference between the simulated temperature and the experimental data is quite small.

As an alternative to the time consuming DOM a fast and sufficiently accurate method based on the formal solution of the radiative transfer equation was developed in [3]. This method is used here to simulate the cooling of a glass disk supported on its edge by a metal support mold. Due to the contact between glass and metal, a boundary condition that describes the exchange of radiative energy not only in the opaque wavelength region but also in the semitransparent region must be taken into account. The selected numerical solution method [3] must be modified to incorporate this kind of boundary condition.

The paper is organized as follows. In Section 2, the axisymmetric model considered in this paper is defined. The geometries of the disk and the mold, the mechanical and thermal equations and the radiative heat transfer model are discussed. To validate this model, the selected numerical solution is compared with values from [17,18] in a specific one-dimensional solution. It turns out that the proposed numerical method for radiative transfer is very close to the experimental data. In Section 3, the method described in Section 2 is applied to the axisymmetric problem of the cooling of the glass disk supported on a metal mold. The results obtained in terms of temperatures and stresses will be discussed. It will be shown that considering internal radiation is very important for simulating glass cooling.

2. Definition of the two-dimensional glass-cooling model

2.1. The geometric model for glass cooling

This paper examines the cooling of a circular disk supported on its edge by a metal mold as described in Fig. 1. The glass domain is defined by $D^g=\{0\leqslant r\leqslant R, 0\leqslant z\leqslant e\}$.

The mold domain is defined by $D^m=\{R_1\leqslant r\leqslant R_2, -E\leqslant z\leqslant 0\}$. Since cooling occurs between time 0 and time t_{max}, $D^g_t=D^g\times\{0\leqslant t\leqslant t_{max}\}$ and $D^m_t=D^m\times\{0\leqslant t\leqslant t_{max}\}$, respectively denote the domains occupied by the glass and the mold over the time. The boundaries of these two domains are denoted by ∂D^g_t and ∂D^m_t respectively. The contact area between the glass and the mold is denoted by ∂D^{gm}_t.

In the present modeling study, it is assumed that:

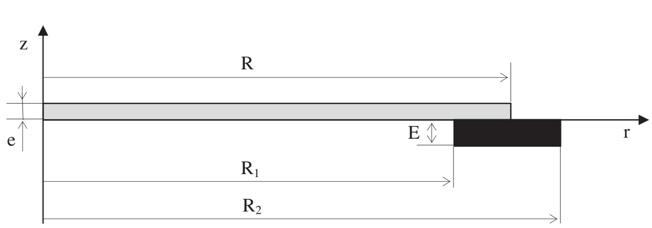

Fig. 1. Axisymmetric model of the circular glass disk supported on its edge by the metal mold.

A1: At the bottom of the mold, there is no displacement in the z-direction:

$$u_z(\bar{r},t)=0, (\bar{r},t)\in\partial D^m_t,\ z=-E.$$

A2: The mold is considered as a thermoelastic body that dilates when heated.

A3: At time $t=0$ s, when tempering begins, temperatures in the glass and in the mold are homogeneous.

A4: For time $t>0$ s, the glass and the mold are cooled by air and the cooling is uniform throughout the glass disk and mold surfaces. Even the bottom of the mold ($z=-E$, $R_1\leqslant r\leqslant R_2$) undergoes cooling.

A5: Mechanically, sliding contact is considered at the interface between the glass and the mold. Thermally, in the contact zone, in addition to accounting for radiative transfer, heat exchange by conduction is considered and modeled by a constant heat transfer.

A6: Gravity is not considered as only the residual stresses field is studied and not on the final shape. In fact, the bending stresses due to gravity are very small and have a negligible impact on the residual stresses.

2.2. Mathematical model for the mechanical behavior

Since the problem is axisymmetric, a cylindrical coordinate system (r,θ,z) is used. The problem does not depend on θ and there is no tangential displacement. The displacement vector in the glass and in the mold is:

$$\bar{u}(\bar{r},t)=\left\{\begin{matrix}u_r(\bar{r},t)\\0\\u_z(\bar{r},t)\end{matrix}\right\}. \tag{1}$$

In the tempering operation, displacements are very small and the linearized strain tensor can be used. Its components are:

$$\varepsilon_{rr}(\bar{r},t)=\frac{\partial u_r}{\partial r}(\bar{r},t),\quad \varepsilon_{\theta\theta}(\bar{r},t)=\frac{u_r(\bar{r},t)}{r},\quad \varepsilon_{zz}(\bar{r},t)=\frac{\partial u_z}{\partial z}(\bar{r},t),$$
$$\varepsilon_{rz}(\bar{r},t)=\frac{1}{2}\left(\frac{\partial u_r}{\partial z}(\bar{r},t)+\frac{\partial u_z}{\partial r}(\bar{r},t)\right),\quad \varepsilon_{\theta z}(\bar{r},t)=0,\quad \varepsilon_{r\theta}(\bar{r},t)=0. \tag{2}$$

In absence of gravity and inertial forces, mechanical equilibrium is:

$$\overline{\nabla}_{\bar{r}}\cdot\boldsymbol{\sigma}(\bar{r},t)=\bar{0},\quad (\bar{r},t)\in D^g_t, \tag{3}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor and $\overline{\nabla}_{\bar{r}}$ the divergence operator in cylindrical coordinates. For an axisymmetric problem, it reduces to:

$$\begin{cases}\frac{\partial\sigma_{rr}}{\partial r}(\bar{r},t)+\frac{\sigma_{rr}(\bar{r},t)-\sigma_{\theta\theta}(\bar{r},t)}{r}+\frac{\partial\sigma_{rz}}{\partial z}(\bar{r},t)=0,\\ \frac{\partial\sigma_{rz}}{\partial r}(\bar{r},t)+\frac{\sigma_{rz}(\bar{r},t)}{r}+\frac{\partial\sigma_{zz}}{\partial z}(\bar{r},t)=0.\end{cases}\quad (\bar{r},t)\in D^g_t, \tag{4}$$

Time is not explicitly present in Eq. (4), but will come into play through the temperature dependence of the material properties and thermal dilatation.

The boundary conditions are the following:

- Due to the axisymmetry, the radial displacement vanishes on axis $r=0$:

$$u_r(\bar{r},t)=0,\quad (\bar{r},t)\in\partial D^g_t,\quad r=0. \tag{5}$$

- Due to the presence of the mold, there is a unilateral contact condition on boundary ∂D^{gm}_t. The displacements of both bodies must satisfy the Signorini condition stating that the bodies cannot interpenetrate and that a contact force only exists when the distance between both bodies vanishes [19].