International Journal of Heat and Mass Transfer 90 (2015) 1090–1097

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A new diffusion for laminar boundary layer flow of power law fluids past a flat surface with magnetic effect and suction or injection

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ARTICLE INFO

Article history:
Received 9 December 2014
Received in revised form 7 May 2015
Accepted 17 July 2015

Keywords:
Nonlinear diffusion
Laminar boundary layer flow
Power law fluid
Suction or injection
Magnetic field
Mass transfer

ABSTRACT

This paper investigates steady laminar boundary layer flow of power law fluids past a flat surface with suction or injection and magnetic effects. A new nonlinear diffusion model is proposed, which takes the effects of power law viscosity on concentration fields into account by assuming that the concentration field is similar to the velocity, with a modified Fick's law of diffusion for power law fluids. The governing strongly nonlinear partial differential equations are reduced into a set of coupled ordinary differential equations and then solved numerically by the shooting technique coupled with the Runge–Kutta scheme and the Newton's method. The effects of the Hartmann number, the suction or injection parameter, the power law index, the concentration power law index, the concentration radio parameter and the generalized Schmidt number on the velocity and the concentration fields are presented graphically and analyzed in detail.

1. Introduction

For half a century, the study of non-Newtonian fluids has attracted much attention to the researchers because of their practical applications, such as thin polymer spreading films, food processing industries, lubrication process, blood circulation, etc. A number of industrially important fluids including molten plastics, polymers, pulps, food and fossil fuels, which may saturate in underground beds, display non-Newtonian behavior. Various constitutive equations have been proposed, such as Ostwald de Waele model [1,2], Binghan model, Casson model, Maxwell model, Jeffrey model [3–5], Olroyd-B model and so on, to explain the behavior of the non-Newtonian fluids. Among these models, the power law model (Ostwald de Waele model) is the most widely used in many fields of applications because of its simplicity. For an incompressible power law fluid past a flat surface, its power-law shear rate-shear stress relation is expressed as

$$\tau = \mu \frac{\partial U}{\partial Y}\left|\frac{\partial U}{\partial Y}\right|^{n-1}$$

where μ is the consistency and n is the power-law index of the fluid. For shear thinning fluids $0 < n < 1$, $n = 1$ corresponds to a Newtonian fluid, and $n > 1$ describes dilatants fluids.

In 1968, the theoretical and experimental studies on the solid–liquid mass transfer from a rotating disk in power law non-Newtonian liquids were reported [6]. Then, the power law non-Newtonian fluid model was used in the bubble drag and mass transfer problem [7]. D'Alessio and Pascal [8] presented a research for steady two-dimensional flow of power law fluid past a non-rotating circular cylinder. In recent years, the basic mechanism of boundary layer flow and mass transfer of power law fluids has been extensively investigated. Jumah and Mujumdar [9] considered pure Darcy-diffusive free convection boundary layer flow of power law fluids with yield stress from a vertical flat plane saturated porous media. The governing equations and boundary conditions are cast into a dimensionless form by similarity transformation and the resulting system was solved by finite difference method. Eldabe et al. [10] studied the thermal-diffusion and diffusion-thermo effects on mixed free-forced convection and mass transfer boundary layer flow for non-Newtonian fluid with temperature dependent viscosity by the aid of Chebyshev finite difference method. Later, Chen et al. [11] developed a theoretical analysis to predict the apparent viscosity of laminar non-Newtonian liquid film in a rotating packed bed. It was shown

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.07.067

Nomenclature and units

a	the suction/injection parameter, [–]
b	the concentration radio parameter, [–]
B_0	a constant, [$\mathrm{kg}^{-1}\,\mathrm{s}^{-2}\,\mathrm{A}^{-1}\,\mathrm{m}^{0.5}$]
$B(X)$	the magnetic induction, [$\mathrm{kg}^{-1}\,\mathrm{s}^{-2}\,\mathrm{A}^{-1}$]
C	the concentration, [m^{-3} mol]
C_w	the concentration of the flat surface, [m^{-3} mol]
C_∞	the concentration of the fluid outside the concentration boundary layer, [m^{-3} mol]
D	the mass diffusivity, [$\mathrm{m}^2\,\mathrm{s}^{-1}$]
f	the similar stream function, [–]
L	the length unit, [m]
M	the Hartmann number, [–]
m	the concentration power law index, [–]
n	the power law index, [–]
Sc	the generalized Schmidt number, [–]
U, V	the velocity components along X and Y directions, respective, [$\mathrm{m\,s}^{-1}$]
X, Y	the Cartesian coordinates along the surface and normal to it, [m]
u, v	the x and y component of the dimensional velocity, [–]
x, y	the Cartesian coordinates along the surface and normal to it (dimensional), [–]
U_∞	the free stream velocity far away from the surface, [$\mathrm{m\,s}^{-1}$]
V_0	a constant, [$\mathrm{m}^{(2n+1)/(n+1)}\,\mathrm{s}^{-1}$]
V_w	the suction or injection velocity across the surface, [$\mathrm{m\,s}^{-1}$]

Greek symbols

ϕ	the dimensional concentration, [–]
ρ	the density, [$\mathrm{kg\,m}^{-3}$]
τ	the shear stress, [Nm^{-2}]
ψ	the stream function, [–]
η	the similarity variable, [–]
δ	the electrical conductivity, [$\mathrm{kg}^{-1}\,\mathrm{m}^{-3}\,\mathrm{s}^3\,\mathrm{A}^2$]
λ	a constant, [$\mathrm{m}^2\,\mathrm{s}^{n-2}\,\mathrm{K}^{-m}$]
γ	a constant, [$\mathrm{m}^2\,\mathrm{s}^{n-2}$]
μ	the kinematic viscosity, [$\mathrm{kg\,m}^{-1}\,\mathrm{s}^{-1}$]

Subscripts

w	for the flat surface of the power law fluids
∞	for the potential flow region

that mass transfer coefficients decreased with increasing viscosity, while the centrifugal force still revealed effective in enhancing mass transfer in viscous media. And a correlation for mass transfer coefficient was proposed and valid for both the Newtonian and non-Newtonian fluids. Afterward, Mahdy [12], Tai and Char [13] and Cheng [14] investigated the Soret and Dufour effects on the free convection boundary layers of non-Newtonian power law fluids over a vertical plate. The porous medium condition, thermal radiation effect and variable wall heat and mass fluxes were considered respectively. Furthermore, Dantas et al. [15] explored determination of the effective radial mass diffusivity in tubular reactors under non-Newtonian power law laminar flow using residence time distribution data.

It was known that the magnetic field was widely used in the semi-conductor crystal growth as well as in the casting technologies to dampen the undesired heat convection and mass transport fluctuations in the melt [16]. And, the effect of suction or injection was extremely important in mass transfer. Some studies showed that the suction effect is twofold; first, it enhances mass transfer and second, it stabilizes the laminar flow by delaying the laminar to turbulent transition [17,18]. In view of these applications, the analysis of boundary layer flow and heat mass transfer of non-Newtonian power law fluids with magnetic field and suction or injection effects was important. Firstly, Ibrahim and Terbeche [19] investigated the boundary layer flow of a power law fluid in the presence of a magnetic field, where the magnetic field is perpendicular to the surface and an electric field perpendicular to the magnetic field. Later, Das and Chakraborty [20] studied the transport characteristics of a non-Newtonian power law fluid flow in a rectangular micro-channel, under the sole influence of electrokinetic forces. Afterward, Vajravelu et al. [21] obtained a numerical solution for the steady MHD non-Newtonian power law fluid flow over a continuously moving surface with species concentration and chemical reaction. Then, Pal and Chatterjee [22] and Hsiao et al. [23] considered variable thermal conductivity and thermophoretic particle deposition effects on MHD free convection flow of non-Newtonian power law fluids from a vertical plate embedded in porous media with Soret and Dufour effects, respectively. A list of the key references in the vast literature concerning the power law fluid model with magnetic field and suction or injection effects was given in Refs. [24–30] among others.

Diffusion equation arises in many sciences such as physics, chemistry, metallurgy, and various engineering problems. The study of diffusion equation in non-Newtonian fluids has attracted much attention. Recently, a considerable attention has been devoted to the problem of how to predict the mass transfer behavior of non-Newtonian fluids. The main reason for this is probably that fluids (such as molten plastics, thin films, pulps, slurries, emulsions, etc.) which do not obey the Newtonian postulate that the stress tensor is directly proportional to the deformation tensor, are produced industrially in increasing quantities [31,32]. Understanding the nature of this force by mathematical modeling with a view to predicting the associated behavior of fluid flow has been the focus of considerable research work. In 1961, Philip proposed a model for some special diffusion process as [33]

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (AB)$$

where θ is the generalized concentration, t is time, A is a constant and B is a function of the concentration gradient $\nabla\theta$. Let $B = \nabla\theta$, we can get the classical diffusion equation (Fick's law). Let $B = |\nabla\theta|^{N-1}\nabla\theta(N > 0)$, we obtain the so-called N-diffusion equation. Furthermore, Wu [34] and Wang [35] investigated a free boundary nonlinear problem for the N-diffusion equation. And the existence, uniqueness of the solutions and analyticity results to the N-diffusion were established. Later, Ikoku and Ramey [36] explored the transient flow of non-Newtonian power law fluids in a homogeneous porous media and a new diffusivity equation was used to describe the behavior of power law fluids. Afterward, Pascal and coworkers [37–40] presented a new convection–diffusion model by consider the nonlinear molecular diffusion for mass transfer in a two-phase system. The modified Darcy's law was derived from the non-linear rheology of the power law fluids. In the model, the molecular diffusion depends nonlinearly on both the concentration and the concentration gradient B which is $B = \theta^m|\nabla\theta|^{N-1}\nabla\theta(N \geqslant 1)$. Lately, Zheng and coworkers [41,42] experimentally and theoretically investigated the diffusion, aggregation and precipitation of