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Numerical investigation of heat transfer in three-fluid stratified flows

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ABSTRACT

Heat transfer in three-fluid stratified flow with prescribed wall temperature is numerically investigated in this article. The level-set method is adopted to capture the two evolving interfaces. Numerical solution of the coupled governing equations within the framework of the finite volume method is employed. Verifications of the model are performed by comparison against exact solution for heat transfer in single fluid flow with prescribed wall temperature, i.e. by setting the three fluids to be identical in the current model. Then, the effects of *We* number, *Fr* number, fluid flow rate ratio, viscosity ratio and thermal conductivity ratio on velocity and temperature fields, and more importantly heat transfer quantified in terms of local Nusselt number (Nu_x), and Nusselt number at fully developed flow region (Nu_f), are studied. The results show the increase of flow rates for fluid 1 (located at the bottom of the channel) and fluid 2 (located at the top of the channel) changes Nu_f at the upper wall, Nu_{fu}, and lower wall, Nu_{fl}, in different manners. Increasing fluid 1 viscosity and thermal conductivity increase Nu_{fu} while decrease Nu_{fl}, respectively. Nu_{fu} decreases and Nu_{fl} increases with the increase of fluid 2 viscosity and thermal conductivity.

1. Introduction

Three-fluid stratified flow invovling heat transfer is widely encountered in many engineering applications. Examples include, but are not limited to, oil and gas systems [1] and heat exchangers [2]. In oil and gas pipes, the produced oil generally flows with gas and water simultaneously and stratified gravitationally. The pressure gradient, gas holdup as well as temperature inside the pipes are important factors for a proper design of the pipes. For heat exchangers, the flow and temperature fields determine the thermal performance and effectiveness. Therefore, knowing the flow and temperature fields inside of these systems is of practical importance.

Three-fluid stratified flow generally occurs at low flow rates of the three fluids in horizontal or slightly inclined pipes [3,4]. In such pipes, the three fluids form three different layers with two distinct interfaces because of gravitational effect. These two interfaces evolve temporally and spatially. With heat transfer involved, the temperature field in the three fluids is affected by their different physical properties apart from the flow field. In particular, this temperature field also changes in response to the movement of the two interfaces in a fully coupled manner. Therefore, central to the modeling of such kind of problem is to capture the two evolving interfaces and at the same time predict correctly the coupled heat transfer in each fluid.

Heat transfer in three-fluid system has been studied extensively. A compact analytical solution of three-fluid heat transfer in a heat exchanger is proposed by Sekulić [5]. The solution considers three-fluid stratified flow for both co-current and counter-current flow configurations. In a separate study, Shrivastava and Ameel [6] proposed a general analytical heat transfer model for three-fluid flow in a heat exchanger. The model was then applied to study the effect of six different parameters on the effectiveness of the heat exchanger [7]. Their results showed that the heat transfer performance was significantly affected by the thermal capacities of the three fluids. The transient temperature distribution in a three-fluid heat exchanger is analytically studied by Bielski and Malinowski [8]. The solution was obtained based on Laplace transforms for a specific case of a constant temperature in one channel. Good agreement was achieved between their results and the semi-analytical solution from Tzou [9]. Singh et al. [10] also studied the transient behavior of three-fluid heat transfer in a heat exchanger. Their results showed that the axial dispersion effect was important at low Peclet number. Chen and Malinowski [11] studied steady state heat transfer in a three-fluid heat exchanger. The analytical results agreed well with their numerical results.

The above mentioned works are limited to the assumption of a uniform velocity distribution of the three fluids. A more general

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Nomenclature

C_p	specific heat capacity (J/kg K)
d	distance (m)
$\vec{f}_\sigma$	surface tension force (Nm^3)
Fr	Froude number
h	thickness of the fluid layer (m)
H	height of domain (m)
$H(\phi)$	smoothed heaviside function
$\vec{g}$	gravity vector (m/s^2)
k	thermal conductivity (W/m K)
L	length of domain (m)
$\hat{n}$	unit normal at the interface
Nu	Nusselt number
p	pressure (Pa)
Pr	Prandtl number
Re	Reynolds number
$\overline{sign}(\phi)$	sign function
S	source term
t	time (s)
$\bar{t}$	pseudo time (s)
T	temperature (°C)
$\vec{u}$	velocity vector (m/s)
We	Weber number
x, y	Cartesian coordinate

Greek symbols

$\delta(\vec{x}-\vec{x}_f)$	Dirac delta function
Γ	interface
ϕ, ϕ'	level set function (m)
θ	dimensionless temperature
ε	interface thickness (m)
μ	dynamic viscosity (kg/m s)
ρ	density (kg/m^3)
σ	surface tension (N/m)
κ	curvature (1/m)
Ω	domain of interest
Φ	general variable in Eq. (19)

Subscripts

f	fully developed flow
fl	lower wall at fully developed flow
fu	upper wall at fully developed flow
in	inlet
ref	reference value
w	wall
x	local
1, 2, 3	different fluids
$*$	dimensionless

consideration requires such a limitation to be removed for an improved prediction with the effect of flow field more appropriately accounted for. Nikodijević et al. [12] studied three-fluid stratified flow and heat transfer in a horizontal channel under an applied magnetic field. They obtained the analytical solution based on the simplified governing equations. Their results showed that the temperature distribution was significantly affected by the physical properties of the fluids. Although with fluid flow considered in Nikodijević et al.'s work [12], only stream-wise velocity variation is considered given the complications involved. The two interfaces separating the three fluids are assumed to have predefined known profiles. This, however, does not completely capture the intimate couplings between the fluids' motions occurring in the process. When the three fluids flow, the two interfaces evolve dynamically. Accordingly, the heat transfer between the three fluids also changes. Therefore, the conservation equations for mass, momentum and energy are naturally fully coupled via appropriate interfacial conditions. Capturing/tracking the evolving interfaces is necessary. However, existing works are lacking in this respect. Therefore, this numerical study is undertaken to incrementally explore such a coupled model. In this article, a numerical model to study three-fluid stratified flow and heat transfer is presented. The level-set model is used to capture the movements of the two interfaces [13] allowing heat transfer between the three fluids to be investigated in fully coupled manner. In this article, the effects of *We* number, *Fr* number, fluid flow rate, viscosity and thermal conductivity on the fluid flow and heat transfer performance are studied. The flow field as well as the temperature field in the three fluids is compared under different parameters. The local Nusselt number, Nu_x, and Nusselt number at fully developed flow, Nu_f at both upper and lower walls are presented.

2. Problem description

The two-dimensional channel of interest with height H and length L is schematically shown in Fig. 1. It consists of three stratified sub-regions, i.e., fluid 1 region (Ω_1), fluid 2 region (Ω_2) and fluid 3 region (Ω_3).The thicknesses of these layers at the inlet are respectively h_1, h_2 and h_3. The three fluids are immiscible in each other. These three regions are separated by interfaces Γ_1 and Γ_2, respectively. Both the upper and the lower walls are maintained at a constant temperature of T_w. Fluid 1, fluid 2 and fluid 3 at temperature of T_1, T_2 and T_3 respectively flow into the channel with inlet velocity of u_1, u_2 and u_3 respectively. As T_w is higher than the fluid temperature, heat is transferred from the heated wall to the fluid adjacent to the walls in the form of convection heat transfer and combined conduction and convection heat transfer between different fluid layers. The fluid flow changes the two interfaces as well as the temperature field simultaneously in a fully coupled manner.

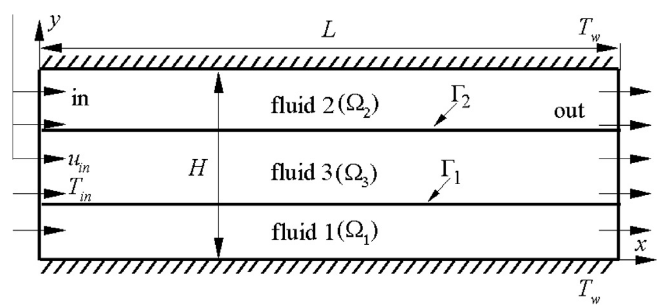

Fig. 1. Schematic of a two-dimensional channel.

3. Mathematical formulation

3.1. Governing equations

In the current work, the evolving interfaces are captured using the level-set method [14]. The level-set function ϕ is defined as the