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A numerical study of nanofluid natural convection in a cubic enclosure with a circular and an ellipsoidal cylinder

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ABSTRACT

In this paper we develop a numerical method and present results of simulations of flow and heat transfer of nanofluids. We consider a heated circular and elliptical cylinder in a cooled cubic enclosure. Natural convection, which drives the flow, and heat transfer are simulated for different temperature differences and enclosure inclination angles. Steady laminar regime is considered with Rayleigh number values up to a million. Al_2O_3, Cu and TiO_2 nanofluids are considered, as well as pure water and air for validation purposes. Properties of nanofluids are considered to be constant throughout the domain and are estimated for different nanoparticle volume fractions (0.1 and 0.2).

In order to simulate nanofluids, an in-house numerical method was developed based on the solution of 3*D* velocity–vorticity formulation of Navier–Stokes equations. The boundary element method is used to solve the governing equations. In the paper, special consideration is given to the estimation of the boundary value of vorticity on an arbitrary curved surface.

The results show highest heat transfer enhancement in the conduction dominated flow regime, where the enhanced thermal properties of nanofluids play an important role. When convection is the dominant heat transfer mechanism, the using nanofluids yields a smaller increase in heat transfer efficiency. Comparison of 2*D* and 3*D* results reveals consistently lower heat transfer rates in the 3*D* case. As the enclosure is tilted against gravity, the flow symmetry around an elliptical cylinder is lost and the overall heat transfer increases.

1. Introduction

Cooling is one of the major challenges in development of efficient devices. Natural convection is used to design many devices, for example, heat exchangers and electronics coolers. Study of natural convection in such devices was started by De Vahl Davies [6], who proposed the now classical problem of a differentially heated cavity. He considered an enclosure, where one wall is heated to a constant temperature and a wall on the opposite side is cooled to a constant temperature. Due to the temperature difference, natural convection develops in the enclosure. Many engineering applications are geometrically more complicated and thus more recently, attention has shifted to enclosures with hot bodies embedded within [9]. Depending on the temperature difference, the natural convection that develops, may be steady and laminar for low temperature differences, while for higher temperature differences transition to turbulence may be observed.

Choice of a working fluid is very important, as its thermal properties determine heat transfer characteristics. As thermal conductivity of water, oil and other working fluids are low, Choi [4] introduced nanofluids. Nanofluid is a suspension consisting of uniformly dispersed and suspended nanometre-sized (10–50 nm) particles in base fluid. Nanofluids have a very high thermal conductivity at a very low nanoparticle concentrations and exhibit considerable enhancement of convection [35]. A wide variety of experimental and theoretical investigations have been performed, as well as several nanofluid preparation techniques have been proposed [32].

Research in the use of nanofluids for natural convection type application has been intensified in recent years [31,18,27]. Hu et al. [11] considered a square enclosure filled with nanofluid and compared experiments and numerical simulations for different nanoparticle concentrations. Oztop and Abu-Nada [21] performed a 2*D* study of natural convection of various nanofluids in partially heated rectangular cavities, reporting that the type of nanofluid is a key factor for heat transfer enhancement. They obtained best results with Cu nanoparticles. The same researchers [2] examined the effects of inclination angle on natural convection

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in enclosures filled with Cu–water nanofluid. They reported that the effect of nanofluid on heat enhancement is more pronounced at low Rayleigh numbers. Hwang et al. [12] studied natural convection of a water based Al_2O_3 nanofluid in a rectangular cavity heated from below. They investigated convective instability of the flow and heat transfer and reported that the natural convection of a nanofluid becomes more stable when the volume fraction of nanoparticles increases. Ho et al. [10] studied effects on nanofluid heat transfer due to uncertainties of viscosity and thermal conductivity in a buoyant enclosure. They demonstrated that usage of different models for estimation of viscosity and thermal conductivity does indeed have a significant impact on heat transfer. Natural convection of nanofluids in an inclined differentially heated square enclosure was studied by Ögüt [20], using polynomial differential quadrature method. Kim et al. [15] studied 2*D* natural convection of air around a circular cylinder within a square enclosure. Sheremet et al. [28] considered natural convection in a 3*D* porous enclosure filled with a nanofluid and compared homogeneous nanoparticle distribution model with a inhomogeneous model. Most of the studies in the literature were done in 2D. In this paper we present development of a 3*D* nanofluid flow simulation algorithm.

Several numerical methods have been proposed for the simulation of nanofluids. Garoosi [8] carried out a numerical study of natural and mixed convection heat transfer of nanofluid in a two-dimensional square cavity with several pairs of heat source-sinks using the finite volume method. Control volume based finite element method was used by Seyyedi et al. [26] to simulate the natural convection heat transfer of Cu–water nanofluid in an annulus enclosure. El Abdallaoui et al. [1] used the lattice Boltzmann method for numerical simulation of natural convection between a decentered triangular heating cylinder and a square outer cylinder filled with a pure fluid or a nanofluid. Elshehabey et al. [7] developed a finite difference method for natural convection in an inclined L-shaped enclosure filled with Cu–water nanofluid that operates under differentially heated walls in the presence of an inclined magnetic field. Kefayati [13] used a finite difference lattice Boltzmann method heat transfer and entropy generation due to laminar natural convection in a square cavity.

In this paper we present a boundary element method based algorithm for simulation of flow and heat transfer of nanofluids. We formulate the Navier–Stokes equations in velocity–vorticity form and couple them with the energy conservation equation. Daube [5] pointed out that the correct evaluation of boundary vorticity values is essential for conservation of mass when using the velocity vorticity formulation. Several different methods were considered for estimation of the vorticity on the boundary. Wong and Baker [33] used a second-order Taylor series to determine the boundary vorticity values explicitly. Daube [5] used an influence matrix technique to enforce both the continuity equation and the definition of the vorticity in the treatment of the 2*D* incompressible Navier–Stokes equations. Liu [16] recognised that the problem is even more severe when he extended it to three dimensions. Lo et al. [17] used the differential quadrature method. Škerget et al. [30] proposed the usage of single domain BEM to obtain a solution of the kinematics equation in tangential form for the unknown boundary vorticity values and used it in 2D. This work was extended into 3*D* using a linear interpolation by Žunič et al. and Ravnik et al. [25] for simple geometries. In this paper we extend these methods for determination of boundary vorticity at an arbitrarily shaped surface.

2. Problem description

A heated cylinder is inserted into an enclosure with four cooled walls. Front and back walls are perfectly insulated (adiabatic). All walls have a no-slip boundary condition applied for velocity. The heat is transferred from the cylinder to the fluid causing density changes that result in buoyancy forces. Natural convection develops – the fluid rises around the cylinder and transports heat towards the cold walls. The heat flux depends on the type of fluid (air, water and nanofluids in this work), the shape of the cylinder and the orientation of the enclosure with respect to gravity.

The centre of the cylinder is located at the centre of the enclosure. The shape of the base of the cylinder is an ellipse with major semi-axis a and minor semi axis b. They are defined as

$$a = 0.2L, \quad b = a\sqrt{1-e^2}, \tag{1}$$

where e is the eccentricity of the ellipse and the length of the cylinder is L. The enclosure is cubic with a volume of L^3. It is tilted with respect to gravity with an angle of γ. The temperature of the cylinder is constant T_h and the temperature of the cold walls is also constant, T_c (see Fig. 1).

3. Governing equations

We consider water based nanofluids, as well as pure water and air for validation and comparisons. Thermophysical properties of solid nanoparticles and all fluids are given in Table 1. Water and nanoparticles are in thermal equilibrium and no slip occurs between them. We assume the nanofluid to be incompressible. Natural convection exhibited by the nanofluids in our simulations is laminar and steady. Effective properties of the nanofluid are: density ρ_{nf}, dynamic viscosity μ_{nf}, heat capacitance $(c_p)_{nf}$, thermal expansion coefficient β_{nf} and thermal conductivity k_{nf}, where subscript *nf* is used to denote effective i.e. nanofluid properties. The properties are all assumed constant throughout the flow domain. Pure fluid properties will be denoted by the subscript *f*.

Dimensionless velocity $\vec{v}$, location vector $\vec{r}$, vorticity $\vec{\omega}$, temperature T and gravity $\vec{g}$ were employed by introducing

$$\vec{v} \to \frac{\vec{v}^\star}{v_0}, \quad \vec{r} \to \frac{\vec{r}^\star}{L}, \quad \vec{\omega} \to \frac{\vec{\omega}^\star L}{v_0}, \quad T \to \frac{T^\star - T_c}{T_h - T_c}, \quad \vec{g} \to \frac{\vec{g}^\star}{g_0}, \tag{2}$$

where, $v_0 = \frac{k_f}{(\rho c_p)_f L}$ is the characteristic velocity and $g_0 = 9.81\ \mathrm{m/s^2}$.

The nondimensional steady velocity–vorticity formulation of Navier–Stokes equations for simulation of nanofluids consists of

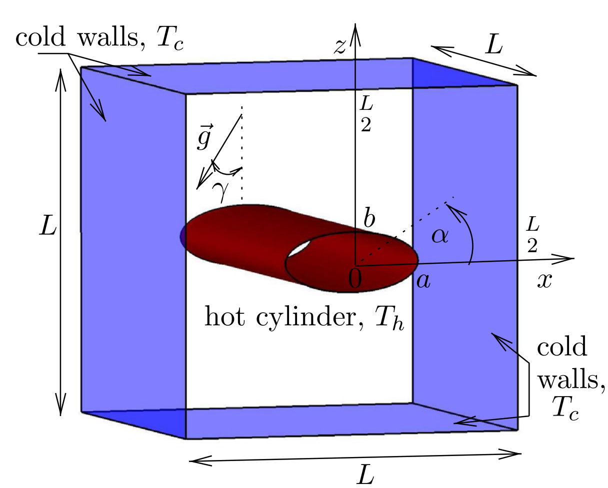

Fig. 1. Computational domain and coordinate system with boundary conditions. The angle α measures a location around the circumference of the cylinder. The angle γ measures the tilt of the enclosure with respect to the gravity vector. The front and back walls ($y = 0$ and $y = L$) are adiabatic.