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Laminar forced convective slip flow in a microduct with a sinusoidally varying heat flux in axial direction

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ABSTRACT

In this study, steady laminar forced convection slip flow in a microtube subjected to an axially varying heat flux is investigated numerically using the finite volume method. The classical Graetz problem is considered, which can be named as micro-Graetz problem for the microscale condition. The viscous dissipation effect is included in the analysis. The problem studied can be also named as micro-Graetz–Brinkman problem. The slip flow regime is considered by incorporating the velocity slip and temperature jump conditions at wall. The effects of rarefaction, the viscous dissipation and the dimensionless amplitude of the axially varying or periodic heat flux on the local and mean Nusselt numbers as well as on the wall and bulk temperatures are obtained for some specific ranges of corresponding parameters. These effects are found to be interactive. It is disclosed that the mean Nusselt number decreases with an increase in the amplitude.

1. Introduction

Rapid progress in microfabrication techniques and wide application areas of microdevices have triggered research interest in heat and fluid flow at microscale. For proper design and reliable operations of these devices, ongoing research interest is very critical and valuable.

The Knudsen number (Kn), the ratio of the gas mean free-path to the characteristic length of the channel, determines the degree of rarefaction and the validity of the continuum approach. For very small values of Kn, the continuum approach is valid. As Kn increases, the mean free path of the gas becomes comparable to the characteristic length of the channel, rarefaction effects become more important and eventually the continuum approach breaks down. It is basically a criterion to classify flow regime of gases. The range of $0.001 \leqslant Kn \leqslant 0.1$ represents the slightly rarefied slip flow regime where the fluid velocity at the wall is not zero and, wall temperature and adjacent fluid temperature are not the same. Slip flow regime is encountered in a wide variety of applications. When analyzing this regime, the Navier–Stokes equations remain valid provided that tangential slip velocity and temperature jump conditions are implemented at the walls.

Colin [1] recently presented an excellent review of investigations on slip flow heat transfer in microchannels, focusing on the Nusselt number dependence on rarefaction (Knudsen number), viscous dissipation (Brinkman number) and axial conduction (Peclet number). For various combinations of hydrodynamic and thermal boundary conditions in various micro-geometries, a considerable amount of studies have appeared in the literature [2–26]. Effects of rarefaction on the heat transfer have been well documented. Some studies have also included effect of viscous dissipation.

Almost all studies on slip flow forced convection in microducts assume constant or uniform heat flux/temperature at wall. However, axially varying thermal boundary conditions at wall are sometimes encountered in practice. Electronics cooling related to periodic micro-electronic heaters/chips, cooling of microreactors, control or enhancement of heat transfer at microscale level and micro-processor chip cooling could be some examples. As a representative one, we note microreactors where axial variation of heat flux, nearly in a sinusoidal manner, exits. For the macroscale case, there are some studies on forced convection in ducts with axially varying thermal boundary conditions [27–38].

To the authors' best knowledge, there is no study on convective heat transfer in microchannels subjected to axially varying heat flux. This article aims at investigating slip flow regime of rarefied gas in a microtube with an axially varying heat flux, taking the

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Nomenclature

A	dimensionless heat flux amplitude, Eq. (6)
Br	Brinkman number, Eq. (10)
D	diameter of the microtube [m]
F	tangential momentum accommodation coefficient
F_t	thermal accommodation coefficient
k	thermal conductivity [W/mK]
Kn	Knudsen number
L	length of the microtube [m]
L^*	dimensionless length of the microtube
Nu	Nusselt number
$\overline{Nu}$	Nu mean Nusselt number
Pr	Prandtl number
q''_o	mean value of the wall heat flux along the microtube [W/m^2]
q''_w	wall heat flux [W/m^2]
r	radial coordinate [m]
R	dimensionless radial coordinate
Re	Reynolds number
r_0	radius of the pipe [m]
Pe	Peclet number
Pr	Prandtl number
T	temperature [K]
u	velocity [m/s]
z	axial coordinate [m]
Z	dimensionless axial coordinate
$\bar{Z}$	dimensionless axial coordinate

Greek symbols

α	thermal diffusivity [m^2/s]
γ	specific heat ratio
λ	molecular mean free path
μ	dynamic viscosity [Pa s]
ρ	density [kg/m^3]
υ	kinematic viscosity [m^2/s]
θ	dimensionless temperature, Eq. (8)
θ_b	dimensionless bulk fluid temperature, Eq. (16)
θ_s	dimensionless fluid temperature at the wall, Eq. (14)
θ_{s-w}	dimensionless temperature jump between the fluid and wall, Eq. (14)
θ_w	dimensionless wall temperature, Eq. (15)

Subscripts

s	fluid properties at the wall
w	wall

effects of velocity slip and temperature jump at the gas–solid interface and viscous dissipation into consideration.

2. Problem description and analysis

Hydrodynamically developed but thermally developing laminar gas flow in a microtube subjected to a sinusoidal heat flux boundary condition is considered (Fig. 1). Thermophysical properties of the fluid are assumed to be constant.

In the analysis, the usual continuum approach is coupled with the two main characteristics of the microscale phenomena: the velocity slip and the temperature jump. The velocity slip is given by [5]

$$u_s = -\frac{2-F}{F}\lambda\frac{\partial u}{\partial r}\bigg|_{r=r_0} \tag{1}$$

where λ ($Kn\,D$) is the molecular mean free path and F is the tangential momentum accommodation coefficient which has a value near unity for most engineering surfaces [5]. The temperature jump at the wall is given by a similar expression

$$T_s - T_w = -\frac{2-F_t}{F_t}\frac{2\gamma}{\gamma+1}\frac{\lambda}{\Pr}\frac{\partial T}{\partial r}\bigg|_{r=r_0} \tag{2}$$

where T_s and T_w are the temperature of the gas at the wall and the wall temperature, respectively. The term F_t represents the thermal accommodation coefficient which depends on the gas and the surface material. Particularly for air, it assumes typical values near unity [5]. For the rest of the analysis, F and F_t will be assumed to be 1.

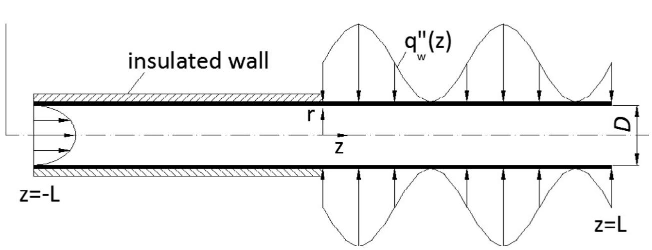

Fig. 1. Schematic of the problem.

The fully developed velocity profile taking the slip flow condition at the wall into consideration is given as [10]:

$$u = \frac{2u_m(1-(r/r_o)^2+4Kn)}{(1+8Kn)} \tag{3}$$

where u_m and Kn are the mean velocity the Knudsen number, respectively.

Under the assumption of local thermal equilibrium, the steady-state energy equation with constant fluid properties is expressed as

$$u\frac{\partial T}{\partial z} = \alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right)+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial r}\right)^2 \tag{4}$$

Here, the third term in the right hand side of the equation is the energy generation due to viscous dissipation. The sinusoidal heat flux boundary condition along the tube wall is given by

$$q''_w(z) = q''_o(1 + A\sin(4\pi z/L)) \tag{5}$$

where A is the dimensionless heat flux amplitude defined as:

$$A = \frac{|q''_w(z)|_{\max}}{q''_o} - 1 \tag{6}$$

The regarding boundary conditions for the energy equation are as follows:

$$z=-L,\quad 0 \leqslant r \leqslant r_o,\quad T = T_e \tag{7a}$$

$$z=L,\quad 0 \leqslant r \leqslant r_o,\quad \partial T/\partial z = 0 \tag{7b}$$

$$r=0,\quad -L \leqslant z \leqslant L,\quad \partial T/\partial r = 0 \tag{7c}$$

$$\begin{aligned} r &= r_o,\quad -L \leqslant z < 0,\quad \partial T/\partial r = 0 \\ r &= r_o,\quad 0 \leqslant z \leqslant L,\quad \partial T/\partial r = q''_w(z)/k \end{aligned} \tag{7d}$$