International Journal of Heat and Mass Transfer 89 (2015) 652–666

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Analytical solutions for three-dimensional steady and transient heat conduction problems of a double-layer plate with a local heat source

Hao-Jie Jiang, Hong-Liang Dai *

State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan University, Changsha 410082, China
Department of Engineering Mechanics, College of Mechanical & Vehicle Engineering, Hunan University, Changsha 410082, China

ARTICLE INFO

Article history:
Received 30 January 2014
Received in revised form 4 May 2015
Accepted 24 May 2015

Keywords:
Analytical solution
Three-dimensional temperature field
Double-layer plate
Local heat source
Steady heat conduction
Transient heat conduction

ABSTRACT

In this paper, analytical solutions for three-dimensional steady and transient heat conduction problems of a double-layer plate with a local heat source are presented, the double-layer structure includes a coating layer and FGM layer. The Poisson method and layer wise (LW) approach are applied to solve the three-dimensional steady heat conduction problem. Meanwhile, to solve the three-dimensional transient heat conduction problem of the double-layer plate, the method of separation of variables (SOV) and LW approach are together applied. The aim of this research is to understand the influences of selected coating material, local heat source and structural parameters on temperature distribution of the double-layer plate, and to guide engineers designing the double-layer structures in thermal environment.

1. Introduction

Double-layer structures are widely used in aerospace, nuclear reactors, internal combustion engines and other fields, and these structures often lie in thermal environment. Therefore, it is very important to investigate the three-dimensional heat conduction problem for the double-layer (Coating/FGM) plate.

For the temperature-related problems of FGM structures, Ding et al. [1] gave a solution of dynamic thermoelastic problem for cylindrical shells. Sadowski et al. [2] carried on theoretical prediction and experimental verification of temperature distributions for FGM cylindrical plates which subjected to thermal shock. Skoczeń [3] obtained FGM structural members via low temperature strain. Without considering energy dissipation, Mallik and Kanoria [4] dealt with the problem of thermoelastic interactions in a FGM structure due to the presence of periodically varying heat sources. By using Green's function technique [5,6], the heat source and heat flux for FGM structures were determined. Utilizing the principle of virtual displacements, Brischetto et al. [7] analyzed a simply supported FGM rectangular plate subjected to thermo-mechanical loadings. Using the homogenization method, Shabana and Noda [8] carried on numerical evaluation of the thermomechanical effective properties for FGM. Shen et al. [9–14] studied a series of mechanical behaviors for FGM structures in thermal environment. Feng and Jin [15] investigated the fracture behavior of FGM plate containing parallel surface cracks which subjected to a thermal shock. Based on the three-dimensional linear theory of elasticity, Li et al. [16] presented free vibration analysis of FGM rectangular plates with simply supported and clamped edges in thermal environment. Using the higher-order shear deformation plate theory, Sun and Luo [17] studied the wave propagation of an infinite FGM plate in thermal environment. By using the infinitesimal theory of magnetothermoelasticity, Dai et al. [18–20] studied thermo-mechanical behaviors of FGM cylindrical and spherical structures. Applying the finite difference method, Hein et al. [21] analyzed the heat conduction problem with spatially varying parameters. Assuming material properties were temperature dependent and graded in the radial direction, Malekzadeh et al. [22] presented a three-dimensional free vibration analysis of the FGM truncated conical shells placed in thermal environment. Due to nonuniform heat supply, Ootao and Ishihara [23] gave a three-dimensional solution for transient thermoelastic problem of a functionally graded rectangular plate with piecewise exponential law.

For effects of coating on resistance of high temperature, Limarga et al. [24] studied high-temperature vibration damping behavior of thermal barrier coating materials. Based on theory and experiment, the tribological properties and wear behaviors [25–33] of variety of coating layers from room temperature to high temperature had been investigated. Based on infrared pyrometry combing with

* Corresponding author at: Department of Engineering Mechanics, College of Mechanical & Vehicle Engineering, Hunan University, Changsha 410082, China. Tel.: +86 731 88664011; fax: +86 731 88711911.

E-mail address: hldai520@sina.com (H.-L. Dai).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.05.094
0017-9310/

Nomenclature

x, y, z	Cartesian coordinates [m]
a, b	plate dimension in x and y direction [m]
h, H	thickness of coating layer and FGM layer [m]
y_0	location of laser point in y direction [m]
y_d	width of local heat source strip [m]
t	time [s]
i, j, k	number of half wave in x, y and z direction
k_1, k_2, k_3	thermal conductivity coefficient in x, y and z direction [W/m K]
k_3^F	heat conduction coefficients of the FGM layer at z direction [W/m K]
k_{im}^0, k_{ic}^0, $(i = 1, 2, 3)$	heat conduction coefficients of metal and ceramic materials at three different directions [W/m K]
T, T_0, ΔT	temperature, room temperature and temperature increment [K]
ΔT_c, ΔT_F	temperature increment for coating layer and FGM layer [K]
ΔT_b	temperature increment at adhesive position [K]
c, ρ	specific heat [J/kg K] and density of the double-layer plate [kg/m^3]
c_m, c_c	specific heat of metal and ceramic materials for FGM layer [J/kg K]
ρ_m, ρ_c	density of metal and ceramic materials for FGM layer [kg/m^3]
F_0	heat flow absorbed by the coating layer [J]
f', f'', f'''	heat flux density [J/m^2]
f	heat generation [J/m^2]
l, n, N	arbitrary discrete layer, functionally graded index and the total number of discrete layer

specific robot spray trajectories, Xia et al. [34] detected and recorded temperature evolution continuously during preheating, spraying and cooling stages. Hodhod et al. [35] determined the effect of different coating types with different thicknesses on the residual load capacities of reinforced concrete loaded column models. Using thermal spray coatings, Matthews et al. [36,37] gave the roles of microstructure in the high temperature oxidation mechanism of Cr_3C_2—NiCr composite coatings. Nguyen et al. [38] studied the influence of cerium oxide argon-annealed coating on the alloy oxidation behavior at 1100 °C. Hernández Rossette et al. [39] investigated the unsteady aerodynamic and aero-thermal performance of a first stage gas turbine bucket with thermal barrier coating and internal cooling configuration. By means of several material characterizations before and after annealing processes, Antonaia et al. [40] discussed the stability of W—Al_2O_3 coating at high temperature. For evaluating the thermal shielding efficiency of intumescent coating, Han et al. [41] deposited a cone calorimeter as heater source which coupled with a thermocouple as detector of the temperature for steel plates. Ohtsu et al. [42] investigated the effect of the heating temperature on the characteristics of the surface layer in a simple treatment process using calcium-hydroxide slurry. Zhang et al. [43] developed a computational model to predict the temperature profile over an organic coating on a metal surface. Applying with detonation-gun spray technology, Kaur et al. [44] studied Cr_3C_2—NiCr coating on T22 boiler steel subjected to high-temperature oxidation and oxidation-erosion environment. Yu et al. [45] showed that the gas temperature could improve the coating deposition efficiency. By a potentially simple, scalable, non-vacuum technique, Srivastava et al. [46] analyzed high temperature oxidation and corrosion behavior of Ni/Ni—Co—Al composite coatings. Abyzov et al. [47] developed a composite material from particles of diamond in a copper matrix of good thermal and mechanical properties. Arizmendi-Morquecho et al. [48] analyzed the high temperature behavior of recycled fly ash cenospheres which deposited thermal barrier coatings. Barshilia et al. [49] found that the multi-functional ZnO coating was reliable for high temperature photothermal conversion applications. Selvakumar and Barshilia [50] presented the state-of-the-art of the physical vapor deposited solar selective coatings. By means of scanning electron microscope [51,52], the surface topography of composite coatings were observed and analyzed. Freni et al. [53] presented novel experimental methods for verification of both hydrothermal and mechanical stabilities of adsorbent coatings. Aydin [54] gave combined effects of thermal barrier coating and blended with diesel fuel on usability of vegetable oils in diesel engines. In short, coatings of structure play great roles in improving the heat resistance.

On the other hand, a few researchers have reported the problems of temperature field. Barik et al. [55] studied stationary plane contact of a heat conducting FGM punch and a rigid insulated half-space. Using Legendre polynomials and Euler differential equations system, Karampour [56] obtained steady temperature distribution for the Poro-FGM spherical vessel. Based on a graded element model, Cao et al. [57] obtained fundamental solution for steady-state heat transfer in FGM. By the methods of Laplace transform and inverse Laplace theorem, Zhou et al. [58] gave exact solutions of the temperature distribution for both the FGM strip and the well stirred fluid. Based on the elasticity theory, Malekzadeh et al. [59] presented the transient analysis of rotating multi-layered FGM cylindrical shells in thermal environment. Applying with Hermitian transfinite element method, Shariyat et al. [60–62] obtained temperature-dependent behaviors for FGM structures under thermal loads. Based on the Lord-Shulman theory, Zhou et al. [63] studied the transient thermoelastic response of FGM rectangular plates. By the methods of Laplace and finite cosine transformations, Ootao and Tanigawa [64–67] studied the three-dimensional transient thermal problems exactly. Using the modified Durbin's numerical inversion method, Keles and Conker [68] solved transient hyperbolic heat conduction in thick-walled FGM cylinders and spheres analytically. By experimental and mixed finite element methods, Aksoylar et al. [69] gave a nonlinear transient analysis for FGM cylinder under blast loads. Jiang and Dai [70] carried out a three-dimensional thermodynamic analysis of simply supported high strength and low alloy rectangular steel plates under laser shock processing. However, as far as we know, analytical solutions of three-dimensional steady and transient heat conduction problem for a double-layer (coating/FGM) plate with local heat source has not been found in literatures.

The aim of this study is to give analytical solutions of three-dimensional steady and transient heat conduction problems for a double-layer (coating/FGM) plate with local heat source. In this study, the Poisson method, LW approach and SOV method are used to solve the three-dimensional temperature field, which could guide engineers designing the double-layer structure adapt to high-temperature environment.

2. Basic formulations of the problem

Consider a double-layer plate with local heat source under a three-dimensional temperature field (as shown in Fig. 1). The Cartesian coordinate system $oxyz$ is set on the top surface $(z = 0)$ of the double-layer plate, here a, b, h and H in Fig. 1 represent the double-layer structure's length, width, thicknesses of material