International Journal of Heat and Mass Transfer 89 (2015) 760–769

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

First and second thermodynamic laws analyses between and inside two rotating solid cylindrical geometries with magnetohydrodynamic flow

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ARTICLE INFO

Article history:
Received 17 January 2015
Received in revised form 12 May 2015
Accepted 26 May 2015

Keywords:
MHD flow
Entropy generation rate
Heat transfer analysis
Co-rotating cylinders
Combined analytical–numerical solution
Variable thermal conductivity

ABSTRACT

Entropy generation rate which is a tool to measure exergy destruction has attracted considerable attention these years. This work is about temperature and entropy generation rate modeling within cylindrical systems using magnetohydrodynamic (MHD) flow. Two solid co-rotating cylindrical geometries with temperature-dependent thermal conductivities and constant, but different, internal heat generations are considered. The inner one is solid and the outer one is hollow. The MHD flow is within the empty space between these cylindrical geometries. Since the middle geometry is considered as fluid flow, the temperature field is coupled with the velocity field. By obtaining the velocity formula as Bessel functions and approximating it with a series form, and employing a combined analytical–numerical solution technique, the temperature formula within all three components of the system can be formulated. Incorporating the obtained temperature field into the provided fundamental entropy generation rates formulas, the local and volumetric averaged entropy generation rates are calculated. Assuming constant thermal conductivity for all materials, completely analytical solution can be achieved for the considered problem. The accuracy and correctness of the combined analytical–numerical solution technique are checked against available analytical solution. After verification, effects of thermophysical parameters such as magnetic field, Brinkman number, different radii, etc. on the velocity field, temperature distribution and entropy generation rates are examined.

1. Introduction

Heat transfer through solids and on their surfaces, which is mainly caused by conduction and convection, have numerous applications both in industries and scientific researches. Although the concept of heat transfer gives us an important tool to quantitatively discuss thermal processes within studied media, it has nothing to do with the quality of the undergoing processes. On the contrary, the second law of thermodynamics brings about a practical tool to qualitatively study thermal processes. The first law of thermodynamics discusses a process from heat transfer point of view which says nothing regarding the entropy generation, i.e., exergy destruction. However, using the second law of thermodynamics one is able to design a system based on minimum entropy generation production. This approach has been discussed thoroughly in a text book by Bejan [1].

Entropy generation within a thermal system is mainly due to heat transfer which occurs by all modes, namely conduction [2–4], convection [5–9], and radiation [10–12]. Other effects may contribute into entropy production such as viscous effects [6–8,13] and magnetic fields [8,14–16]. Bejan's work [1,17] opened the door to other researchers to reconsider heat transfer phenomena from the second law of thermodynamics point of view. Thereafter, many scholars started to reexamine thermal processes from second law of thermodynamics' perspective.

Magnetohydrodynamics (MHD) has been a focus of intense research for a long time due to its vast importance in numerous fields ranging from several natural phenomena such as geophysics and astrophysics to many engineering applications such as plasma confinement, liquid metal, electromagnetic casting and so on [15,18]. Many studies have been conducted regarding the entropy generation due to MHD flow. Mahmud and Fraser [19] studied MHD free convection and entropy generation in a square porous cavity. They found that magnetic field has important effect on the thermal performance and entropy generation of the system. Ibáñez et al. [20] investigated the effect of magnetic field on the velocity, temperature and entropy generation of the flow between two infinite parallel walls of finite electrical conductivity. Effect of slip on the entropy generation in MHD flow over a rotating disk was investigated by Arikoglu et al. [14]. The solution was carried

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.05.101
0017-9310/

Nomenclature

a_1	slope of the thermal conductivity-temperature curve for inner solid material, K^{-1}
a_2	slope of the thermal conductivity-temperature curve for outer solid material, K^{-1}
B_0	magnetic field, T
Br	Brinkman number ($Pr \cdot Ec$)
c_f	specific heat of fluid, $J\,K^{-1}$
Ec	Eckert number
k_1	reference thermal conductivity for inner solid material, $W\,m^{-1}\,K^{-1}$
k_2	reference thermal conductivity for outer solid material, $W\,m^{-1}\,K^{-1}$
k_f	thermal conductivity of fluid, $W\,m^{-1}\,K^{-1}$
k_{f1}	ratio of fluid thermal conductivity to inner solid material reference thermal conductivity
k_{f2}	ratio of fluid thermal conductivity to outer solid material reference thermal conductivity
M	Hartmann number
Pr	Prandtl number
Q_1	dimensionless volumetric internal heat generation rate for the inner solid material
Q_2	dimensionless volumetric internal heat generation rate for the outer solid material
$\dot{q}_1$	volumetric internal heat generation rate for the inner solid material, $W\,m^{-3}$
$\dot{q}_2$	volumetric internal heat generation rate for the outer solid material, $W\,m^{-3}$
R	dimensionless radius
R_2	dimensionless interface radius between the fluid and outer cylindrical geometry
R_3	dimensionless outer radius of the outer cylindrical geometry
r	radius, m
r_1	interface radius between the fluid and inner cylindrical geometry, m
r_2	interface radius between the fluid and outer cylindrical geometry, m
r_3	outer radius of the outer cylindrical geometry, m
T	temperature, K
T_0	reference temperature, K
T_1	temperature of the inner solid material, K
T_2	temperature of the outer solid material, K
T_f	temperature of the fluid, K
T_{out}	temperature at outer side of the outer cylinder, K
U	dimensionless tangential velocity of the fluid
u_θ	tangential velocity of the fluid, $m\,s^{-1}$
u_1	peripheral velocity of the outer surface of the inner cylinder, $m\,s^{-1}$
u_2	peripheral velocity of the inner surface of the outer cylinder, $m\,s^{-1}$

Greek symbols

α_1	dimensionless slope of the thermal conductivity-temperature curve for inner solid material
α_2	dimensionless slope of the thermal conductivity-temperature curve for outer solid material
μ_f	dynamic viscosity, $kg\,s^{-1}\,m^{-1}$
θ	dimensionless temperature
θ_1	dimensionless temperature of the inner solid material
θ_2	dimensionless temperature of the outer solid material
θ_f	dimensionless temperature of the fluid
θ_{out}	dimensionless temperature at outer side of the outer cylinder
ω_1	angular velocity of the inner cylinder, $rad\,s^{-1}$
ω_2	angular velocity of the outer cylinder, $rad\,s^{-1}$
ω_r	ratio of angular velocity of the outer cylinder to inner cylinder
σ	electrical conductivity of fluid, $S\,m^{-1}$

out by differential transformation method (DTM). Ibáñez and Cuevas [21] considered entropy generation due to MHD flow in a rectangular microchannel. They assumed thermally fully developed flow and incorporated conduction heat transfer of lower and upper walls into the thermal and entropy generation analyses. Yazdi et al. [22,23] have opted in favor of entropy generation in MHD flow within open parallel microchannels. Liu and Lo [24] studied entropy generation within mixed-convection flow in vertical channels in the presence of magnetic field. The well-known DTM was used. In an interesting paper, Turkyilmazoglu [16] analytically derived temperature and velocity fields of the MHD nanofluids flow over a permeable stretching/shrinking surface. Mahian et al. [25] analyzed velocity, temperature and entropy generation rate on MHD flow between two concentric rotating cylinders. Sundar et al. [26] conducted an experimental study on the forced convection heat transfer and friction factor in a tube with magnetic Fe_3O_4 nanofluid. Rashidi et al. [27] studied entropy generation in transient MHD flow over a stretching rotating disk with homotopy analysis method. Using artificial neural network and particle swarm optimization algorithm they have also accomplished minimum entropy generation analysis. Sheikholeslami et al. [15] investigated MHD effects on Al_2O_3–water nanofluid flow and heat transfer in a semi-annulus enclosure using Lattice Boltzmann Method. Butt and Ali [28] analytically analyzed the effects of entropy generation in MHD flow over a permeable stretching sheet embedded in a porous medium in the presence of viscous dissipation. Mahian et al. [29] analytically analyzed velocity, temperature and entropy generation rate of MHD flow within a vertical annulus using TiO_2/water nanofluid. Chinyoka and Makinde [30] investigated the inherent irreversibility in an unsteady hydromagnetic generalized Couette flow. The electrical conductivity of the fluid is assumed variable and induced electric field is incorporated into the modeling. In another interesting study, Mahian et al. [31] analytically studied temperature and entropy generation rate of a MHD nanofluid flow between two cylinders where one or both of the cylinders rotate. The inner and outer surfaces were considered isothermal and effects of some physical parameters such as Hartmann and Brinkman numbers on the temperature and entropy generation rate were analyzed. Rashidi et al. [32] considered the second law of thermodynamics to analyze MHD nanofluid flow over a porous rotating disk. Mahian et al. [33] analytically modeled velocity, temperature and entropy generation rate due to mixed convection between two vertical isothermal cylinders with magnetic field.

From the other side, studies within pure conductive media have attracted considerable attentions these years. Ibanez et al. [34] analyzed the temperature distribution and entropy generation within a solid wall and viscous flow between two parallel walls. Kolenda et al. [35] extended the pure conductive part of the work of Ibanez et al. [34] to two and three dimensional modellings. They have pointed out that using internal heat generation within the wall material, minimization of the entropy generation is possible [35]. Al-Qahtani and Yilbas [36] investigated temperature and entropy generation within a semi-infinite specimen. They