



Chebyshev collocation spectral method simulation for the 2D boundary layer flow and heat transfer in variable viscosity MHD fluid over a stretching plate



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ABSTRACT

In this work, the study of fluid flow and heat transfer characteristics of a magnetohydrodynamic (MHD) fluid over a stretching plate in steady state in the presence of magnetic field is conducted. The viscosity of the fluid is assumed to vary as a linear function of temperature given by the following relation: $\mu = \mu^*[a + b(T_w - T)]$. For the research of fluid flow and heat transfer in boundary layers, one usual way to treat the governing partial differential equations (PDEs) is to transform the PDEs into ordinary differential equations (ODEs). While, in the present work, the governing PDEs are kept their originals and only transformed into dimensionless forms. After that, the equations are solved directly by Chebyshev collocation spectral method (CSM) and the effects of more factors on heat transfer and fluid flow can be investigated. Effects of the variable viscosity parameter b , the Hartmann number Ha , and the Prandtl number Pr on the fluid flow and heat transfer characteristics are investigated.

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1. Introduction

The study of boundary flow and heat transfer has received considerable attentions because of its increasing industrial applications and important bearings on several technological processes, such as the manufacture, the drawing of plastics and rubber sheets, the metal and polymer extrusion processes, the glass-fiber, the paper production, and the cooling of metallic sheets in a cooling bath, etc. Blasius [1] firstly investigated the boundary layer flow on a flat plate and employed a similarity transformation in 1908. Different from Blasius, Sakiadis [2,3] considered the boundary layer flow on a moving flat plate in a quiescent fluid and obtained numerical solution of the problem. In 1989, Chappidi and Gunnerson [4] pointed out that there are two sets of boundary value problems formulated by the boundary conditions of $U_\infty > U_w$ and $U_w > U_\infty$. Later, Afzal et al. [5] combined the Blasius ($U_w = 0$) and the Sakiadis ($U_\infty = 0$) problems successfully and obtained a single set of equations. Pohlhausen [6] extended the work of Blasius [1] to account for heat transfer on a flat plate. Thereafter, the problem of flow and heat transfer over a stretching

surface has been investigated and discussed by many researchers [7–10].

As for an electrically conducting fluid, the imposed magnetic field may play an important role on controlling momentum and heat transfers in the boundary layer MHD flow over a stretching sheet [11,12]. The applied magnetic field can make the streamlines steeper and the boundary layer thinner significantly [13]. Thereafter, the study of the MHD boundary layer flow over a stretching sheet has been carried out by many researchers [14–19]. Moreover, the study of the flow and heat transfer characteristics of electrically conducting fluids has attracted considerable interest due to its applications in the process of purification of molten metals from non-metallic inclusions [20–22]. In most studies mentioned above, the fluid viscosity was assumed to be uniform in the flow region. From the physics view, it is known that, with the rise of temperature, the viscosity coefficient decreases in case of liquids whereas increases in case of gases. Abel et al. [23] studied the viscoelastic fluid flow and heat transfer over a stretching sheet with variable viscosity. Mukhopadhyay [24] investigated a hydro-magnetic flow over a heated stretching sheet with variable viscosity in the conditions of different Prandtl number and magnetic parameter.

The governing partial differential equations (PDEs) mentioned in the above reports are usually converted into nonlinear ordinary

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Nomenclature

$\mathbf{A}_U$	matrix defined in Eq. (28)
$\mathbf{A}_\theta$	matrix defined in Eq. (29)
a, b	fluid viscosity variation parameters
$\mathbf{B}_U$	matrix defined in Eq. (28)
$\mathbf{B}_\theta$	matrix defined in Eq. (29)
B_0	applied magnetic field, T
$\mathbf{C}_U$	matrix defined in Eq. (28)
$\mathbf{C}_\theta$	matrix defined in Eq. (29)
$\mathbf{D}^{(1)}$	first order derivative matrices
$\mathbf{D}^{(2)}$	second order derivative matrices
Ha	Hartmann number
L	characteristic length, m
N	number of grid points
Pr	Prandtl number
s	collocation point
T	temperature, K
t	time, s
U	dimensionless velocity component in x-direction
U_0	Reference velocity
V	dimensionless velocity component in y-direction
u	velocity component in x-direction, m/s
v	velocity component in y-direction, m/s

X, Y	dimensionless Cartesian coordinates
x, y	Cartesian coordinates, m

Greek symbols

κ	thermal diffusivity, m^2/s
ρ	density, kg/m^3
ν	kinematic viscosity, m^2/s
μ	dynamic viscosity, $\text{kg}/(\text{m s})$
σ	electrical conductivity, $1/(\Omega \cdot \text{m})$
θ	dimensionless temperature

Superscripts

n	time step
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Subscripts

w	condition at the surface
∞	condition in the ambient medium
x, y	coordinate axis
i, j	grids indexes for x- and y-direction
max	maximum value

differential equations (ODEs) using a similarity transformation by introducing a set of similarity variables [25–27] or picking up the method of Lie group method [19,28–31]. Instead of adopting similarity transformation to simplify the non-linear PDEs into the ODEs as a sort of the one dimensional analysis, another feasible way to deal with the original governing equations is to transform them into a system of non-dimensional differential equations by introducing the variable transformation. Thus the type of mathematical model of the governing equations remains non-linear partial differential, therefore, researchers can solve them using different numerical methods or analytical methods. However, these methods are not prevalent up to now. Sharma solved the unsteady MHD free convection heat and mass transfer of viscous fluid flowing through a porous regime adjacent to a moving vertical semi-infinite plate by element free Galerkin method [32]. Nasser [33] solved the PDEs in 2D boundary layer flow and heat transfer with the help of implicit-Chebyshev pseudospectral method, Newton method was adopted in their iteration.

In order to reach a grid-independent solution, numerical methods such as the finite volume method (FVM) or the finite difference method (FDM) are often used to solve the PDEs with a large number of nodes. To overcome the drawbacks of the low order methods, the spectral method can yield higher accuracy for a smooth solution with far fewer nodes and therefore less computational time. The collocation spectral method (CSM) is a kind of the spectral method which makes use of Chebyshev polynomials in the non-periodic directions. The CSM has been widely applied in computational fluid dynamics [34–36], radiative heat transfer [37–39], and heat transfer problems involving coupled radiation and conduction [40].

The main objective of this work is to investigate the effects of the variable viscosity parameter b , and the Prandtl number Pr on the boundary layer flow and heat transfer over a semi-infinite flat plate under the influence of a magnetic field. Different from the previous studies, the two-dimensional analysis is performed rather than adopting the similarity transformation as a sort of the one-dimensional analysis. Moreover, the PDEs are solved directly by the CSM. Numerical results are discussed for various values of physical parameters.

2. Problem formulation

2.1. Physical model

The steady laminar, mixed convective flow of an electrically conducting viscous fluid over a continuous stretching semi-infinite flat plate (see Fig. 1) is considered. The x axis is in the direction of the stretching plate, and the y axis is perpendicular to the plate. The velocity components in the directions of x and y are u and v respectively. A magnetic field B_0 is imposed parallel to the y axis which is normal to the flow direction.

The properties of the fluid are assumed to be constant except for the dynamic viscosity. Viscous dissipation and Ohmic dissipation are neglected.

2.2. Governing equations

The boundary layer form of the governing equations can be written as:

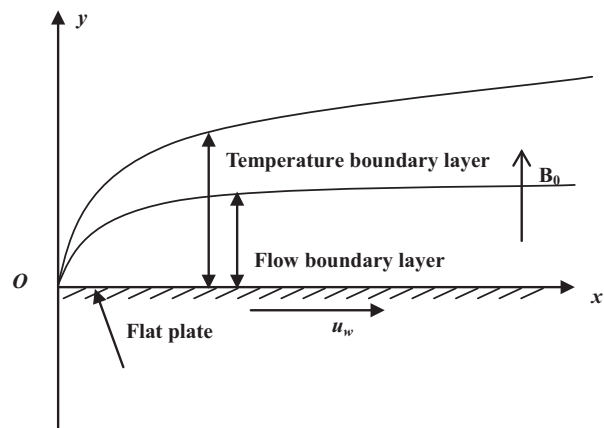


Fig. 1. Physical model and coordinate system.

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