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Solution of the inverse jet in a crossflow problem by a predictor–corrector technique

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ABSTRACT

A predictor–corrector method, developed early, was modified to suit the inverse jet flow in a crosswind problem. The methodology was tested against both numerical and experimental data. The jet was generated by heating compressed air with a velocity range of 0–5 m/s and temperatures up to 425 K. The method attempts to predict the jet velocity, temperature, inlet axial location, and elevation with a self imposed limitation on the number of sample points within the domain. The case where all four of the parameters are unknown led to inaccurate and unacceptable results with 9 sample points. The thermal self-similarity of the problem results in an infinite number of solutions to the problem, with no possibility of narrowing the solution count without more information. Knowing the elevation of the jet results in a maximum error of 9%, but typically much better. Experimental tests indicate the methodology is sensitive to error in the sampling data with a few cases reaching an error over 20%. This technique may be extended to applied areas such as exhaust stacks and fuel injection systems.

1. Introduction

Thermal-fluid systems often create situations in which the engineering problem is an inverse heat transfer problem. These problems often have limited physical access, very limited to no boundary condition knowledge, and/or limited domain information.

For example, the temperature distribution at the wall of an optical fiber drawing furnace is difficult to measure directly due to shape, inaccessibility, and high temperatures. The center of the furnace is easily accessible, where the temperature distribution on an inserted rod may be measured. This leads to the inverse heat transfer problem to obtain the wall temperature distribution that gives rise to the measured rod temperature distribution. Issa et al. [1] developed a regularization technique, utilizing this approach, to determine the wall temperature distribution.

The inverse convection problems have been gaining popularity as of late. Prud'homme and Nguyen [2] solved transient inverse convection problems with a single sensor utilizing the conjugate gradient method, but the sensor needed to be moved closer to the boundary layer as the Rayleigh number increased. A partially adiabatic enclosure with heat loss was solved. Liu et al. [3] determined the thermal profiles in a slot vented enclosure, also utilizing the conjugate gradient approach, requiring tens of iterations to achieve less than 1% error. Hong et al. [4] solved the inverse problem of a differentially heated enclosure with constant wall temperatures. They demonstrated that using the conjugate gradient method required at least nine sample points to resolve the heat flux into the enclosure. The conjugate gradient method is a popular method for solving inverse convection problems and is used a number of other works (e.g. [2–6]).

A different approach is that of the artificial neural network to solve the inverse heat transfer problem. Both Ghosh et al. [7] and Kumar and Balaji [8] successfully applied the technique. Ghosh et al. [7] solved for the heat conduction in a plate. While Kumar and Balaji [8] solved a similar differentially heated enclosure as [4]. Although requiring more sample points than [4] the technique once trained is non-iterative and thus, typically quicker.

Another example is the inverse plume in a crossflow problem. The problem entails solving for the plume boundary conditions, utilizing limited domain knowledge. A novel predictor–corrector method was developed by VanderVeer and Jaluria [9] to solve such a problem. The method requires a specific pattern of known points to match exactly against a set of simulations to predict the inverse solution. The specific pattern was optimized to require the least number of data points for plume in a crossflow problem [10]. With zero error in the data, a minimum of three known points was possible. However, small amounts of error, as is usually the case, would require the known point count to increase to at least five.

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Nomenclature

$\boldsymbol{r}$	vector location of sampled points
a	number of sample locations used in the predictor stage
b, m, Γ_0, Γ_1, Γ_2	model parameters
C_1, C_2, $C_{1\epsilon}$, C_μ, σ_k, σ_ϵ	k–ϵ model coefficients
d	number of simulations
E	thermal energy
F	minimization function for temperature
G	minimization function for velocity
k, ϵ	turbulence kinetic energy, dissipation rate
l, I	turbulence length scale and intensity
n	number of sample locations
P_{rt}	turbulent Prandtl number
T	temperature
U	free stream velocity
X, Y	normalized coordinates
x, y	coordinates

Greek symbols

$\boldsymbol{\Delta}$	relative difference between the first sampled point and other sampled points
$\boldsymbol{\delta}$	vector distance between the actual sampled location and the current test location
λ	thermal conductivity
μ	dynamic viscosity
μ_t	eddy viscosity
ϕ	normalized temperature $\phi = \frac{T-T_\infty}{T_S-T_\infty}$
ρ	density
ε	error associated with the inverse convection method at a location with given sampled data

Superscripts

$*$	predictor stage, alternative heat flux equation

Subscripts

0, 1, 2	sample point indexes
∞	free stream
A, B	data set A,B
i, j, k	index
mod	modified
P	predicted
S	source

The present work is the logical progression of the inverse plume in a crossflow problem, that is the inverse jet in a crossflow problem. The inverse jet in a crossflow problem has many more practical applications, such as exhaust stacks and fuel injection systems. The previous technique will be modified to meet the needs of the new problem.

2. Experimental system

The experiment consists of a wind tunnel with a surface level jet located within the test section. The jet uses compressed air flowing through straighteners to achieve a velocity U_S and is heated to temperature T_S. The jet is subjected to a perpendicular crossflow velocity U_∞. Fig. 1 is a diagram of the wind tunnel and the jet, with dimensions in millimeters.

The wind tunnel test section dimensions are $54.5 \times 305 \times 254$ mm. The maximum velocity of the wind tunnel is 5.0 m/s. The jet is heated by electric cartridge heaters (Omega AHP-7561) with a maximum temperature of 425 K, due to material limitations of the wind tunnel. The X-direction is directed downstream of the wind tunnel with the zero at the center of the jet. The Y-direction is in the direction of the jet and is zero at the surface of the wind tunnel. Due to the large aspect ratio of the wind tunnel $\approx (5:1)$, the flow is assumed to be two-dimensional.

The free stream velocity is determined by a Pitot-Static tube attached to a NIST traceable differential pressure sensor from Omega(PX655-0.1DI). The pressure sensor has a full scale reading of 2.54 mm of water and is accurate to 0.05% of full scale. This results in a maximum error of 3% in the calculated velocity. The jet velocity is determined utilizing a rotameter and verified using a Pitot-Static tube attached to the same previously described pressure sensor. This results in the same amount of error in the jet velocity.

The temperature of domain is measured using a K-type thermocouple mounted to an X–Y traversing stage. Sampled data over the course of several days indicate repeatability of the experiment to within 2%.

3. Numerical simulations

The simulations were all performed using Ansys Fluent [11]. The Navier–Stokes equations were solved using a three-dimensional, steady state, realizable k–ϵ model with enhanced wall effects. Conjugate heat transfer effects are modeled. The free stream Reynolds number Re_{infty} is of order 6×10^3, while the jet Reynolds number Re_S is between 10^3 and 10^4. The Rayleigh number Ra is of order 10^7.

The governing equations are expressed below:

$$u_i = \overline{u_i} + u_i' \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0 \tag{2}$$

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(2S_{ij} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right) - \rho\overline{u_i' u_j'}\right] \tag{3}$$

$$\frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial x_i}[u_i(\rho E + P)] = \frac{\partial}{\partial x_i}\left[\left(\lambda + \frac{C_p \mu_t}{P_{rt}}\right)\frac{\partial T}{\partial x_i}\right] \tag{4}$$

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho k u_j) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + \frac{\partial u_j}{\partial x_i}\left(-\rho\overline{u_i' u_j'}\right) - g_i \frac{\mu_t}{\rho P_{rt}}\frac{\partial \rho}{\partial x_i} + \rho\epsilon \tag{5}$$

$$\frac{\partial}{\partial t}(\rho\epsilon) + \frac{\partial}{\partial x_j}(\rho\epsilon u_j) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial x_j}\right] + \rho C_1 S\epsilon - \rho C_2 \frac{\epsilon^2}{k + \sqrt{\nu\epsilon}} - C_{1\epsilon}\frac{\epsilon}{k}C_{3\epsilon} g_i \frac{\mu_t}{\rho P_{rt}}\frac{\partial \rho}{\partial x_i} \tag{6}$$

$$-\rho\overline{u_i' u_j'} = 2\mu_t S_{ij} - \frac{2}{3}\delta_{ij}\left(\rho k + \mu_t \frac{\partial u_k}{\partial x_k}\right) \tag{7}$$

The constants for the turbulence model are [12,13]:

$$C_{1\epsilon} = 1.44, \quad C_2 = 1.9, \quad \sigma_k = 1.0, \quad \sigma_\epsilon = 1.2, \quad P_{rt} = 0.85 \tag{8}$$