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Isogeometric configuration design optimization of heat conduction problems using boundary integral equation

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ABSTRACT

The shape variation of a domain naturally results in both shape and orientation variations, so called configuration variation, when employing a boundary integral equation (BIE) method. A configuration design sensitivity analysis (DSA) method is developed for steady state heat conduction problems using the boundary integral equations in an isogeometric approach, where NURBS basis functions in a CAD system are directly utilized in the response analysis. Thus, we can accomplish a seamless incorporation of exact geometry and the higher continuity into a computational framework. To enhance the accuracy of configuration design sensitivity, the CAD-based higher-order geometric information such as normal and tangent vectors is exactly embedded in the design sensitivity expressions. The necessary velocity field for configuration design obtained from the NURBS is analytically decomposed into shape and orientation velocity fields. It is shown to be essential to consider orientation variations and significant for accurate configuration sensitivity through comparison with finite differencing conventional BIE method. The developed isogeometric configuration DSA method turns out to be accurate compared with the analytic solution and the conventional DSA method. During the optimization, a mesh regularization scheme is employed to avoid excessive mesh distortion, which comes from significant design changes.

1. Introduction

Ever since the framework of the isogeometric analysis (IGA) method is established by Hughes et al. [1], the isogeometric method that employs the same basis functions as used in the CAD model has shown many advantages over the standard finite element method (FEM). The geometric approximation inherent in the FEM mesh could end up in accuracy problems in response analysis and more adversely in design sensitivity analysis. Besides, the isogeometric method has a major feature such as the CAD based parameterization of field variables in an isoparametric manner. Thus, it requires no further communication with the CAD systems during the refinement processes. In applying the IGA to shape design optimization problems, accurate design sensitivity analysis (DSA) is essential. Based on the shape DSA theory [2], Cho and Ha [3] showed the applicability and accuracy of the isogeometric shape DSA method for the displacement and stress measures. Qian [4] derived shape design sensitivity equation with respect to positions and weights of NURBS control points. Wall et al. [5] showed a structural shape optimization framework based on the isogeometric analysis approach. In addition to the benefits of IGA, the isogeometric DSA has the following advantages: *First*, it provides more accurate sensitivity of complicated geometries including higher order effects such as curvature as well as normal and tangential vectors information. The NURBS functions of higher continuity offer a much more compact representation of response and sensitivity of structures than the standard finite element functions do, yielding better accuracy even at the same polynomial order. *Second*, it vastly simplifies the design modification of complicated geometry without communication with the CAD description. Since the NURBS basis functions are used in both isogeometric response and sensitivity analyses, design modifications are easily obtainable using the adjustment of control points which represent the geometric model. The design velocity field, defined as a mapping rate between the original and perturbed domains, plays an important role in computing configuration design sensitivity coefficients. For the computation of design velocity field, a combination of isoparametric mapping and boundary displacement methods is ideal [6]. When using a conventional FEM, the inter-element continuity of design space is not guaranteed and thus curvature, normal vector, and tangential vector are not

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Nomenclature

h_c	convection coefficient	w_n	fundamental solution for normal flux intensity
$\mathbf{n}$	outward normal vector	$\mathbf{V}$	shape design velocity
$\mathbf{s}$	tangential vector	V_n	normal velocity
Q	internal heat generation	V_s	tangential velocity
$\mathbf{q}$	heat flux	Ω	domain
T	temperature field	Γ	boundary
T_∞	ambient temperature	γ	thermal conductivity
η	fundamental solution for temperature	κ	curvature

accurate enough. On the other hand, in the isogeometric DSA, these are continuous over the whole design space so that accurate shape sensitivity is obtainable.

A boundary integral equation (BIE) method for potential problems was developed by Jaswon [7] and Symm [8] as a pioneering work and extended to elasticity problems by Cruse [9]. Since then, the BIE method has extended its applications to heat conduction, acoustic, and crack propagation problems by means of a powerful and alternative numerical method. However, a singularity problem arises due to the singular fundamental solution expressed as Green functions. The difficulty of dealing with these singularities has been a main issue in the application of BIE method in various engineering problems, which had naturally led to several integration schemes to handle the singular integrals. The computation of Cauchy Principle Value (CPV) for strong singular integrals was proposed by Guiggiani and Casalini [10] as a direct approach and a rigid body method was developed by Brebbia [11] as an indirect approach. Liu and Rudolphi [12] shows the integral identities for fundamental solutions without the computation of CPV. Meanwhile, a weakly singular integration can be implemented based on the transformation method by Telles [13]. Recently, a BIE method employing the isogeometric approach was developed together with the collocation method to precisely locate the field and the source points [14].

In the general expression of design variations, the shape variation of a domain naturally results in both shape and orientation variations in the BIE method. Therefore, the tangential and the normal design velocity fields should have been taken into account in the BIE-based shape design sensitivity analysis (DSA) method that was developed several decades ago. Using the BIE and adjoint variable method in continuum approach, Choi and Kwak derived shape DSA methods for the self-adjoint elliptic boundary value problems [15] and the applications for general stress constrained problems in terms of tangential and normal design velocity fields [16]. The BIE-based DSA method was further applied to the shape optimization for many engineering problems such as heat conduction, acoustic, and so on. Also, an extension to isogeometric shape optimization was performed for elastic problems [17].

There are very few studies conducted on the shape optimization of heat conduction problems. Tortorelli et al. derived the shape design sensitivity for nonlinear transient thermal systems using a Lagrange multiplier method [18] and the adjoint method [19]. Sluzalec and Kleiber [20] employed the Kirchhoff transformation to derive the shape design sensitivity expressions for linearized heat conduction problems using an adjoint variable approach. Li, et al. [21] performed a shape and topology optimization of heat conduction problems using an evolutionary structural optimization method. Dems et al. derived first-order sensitivity equation of heat conduction problem with respect to material property, external boundary, and internal interface. They obtained the optimal design for various problems [22] including steady state conduction with radiation [23]. Wu [24] employed the body-fitted grid generation scheme to generate curvilinear grids and determined shape profiles of heat conduction problem using the finite volume method. Ha and Cho [25] formulated a level set based design optimization method for heat conduction problems, which facilitates topological shape variations. Recently, Yoon, et al. [26] developed isogeometric shape optimization of heat conduction problem using accurate shape sensitivities obtained from the exact normal vector and curvature by NURBS.

The remainder of this paper is organized as follows; *in Section 2*, we describe the construction of NURBS basis functions, which may have up to (p-1) continuous derivatives across element boundaries where p is the order of the underlying polynomial and explain isogeometric BIE method based on the NURBS. In Section 3, we derive the isogeometric BIE configuration design sensitivity considering the shape and the orientation variations. We discuss the expressions of design velocity fields, where the geometric effects seem to have profound effects on the orientation sensitivity. *In Section 4*, demonstrative numerical examples are presented to verify the accuracy of the isogeometric sensitivity by comparing it with the analytic or the conventional BEM solutions. *Finally*, we draw conclusions, which present the importance of isogeometric approach and the exactness of shape and orientation design velocity fields represented by NURBS.

2. Isogeometric boundary integral equation

2.1. NURBS basis function

In the IGA, the solution space is represented in terms of the same basis functions as used in describing the geometry. The isogeometric analysis has several advantages over the conventional finite element analysis (FEA): *geometric exactness* and *simple refinements* due to the use of NURBS basis functions which are based on B-splines. Consider a knot vector $\mathbf{\Xi}$ in one-dimensional space, which includes the set of knots ξ_i in a parametric space.

$$\mathbf{\Xi} = \{\xi_1, \xi_2, \cdots, \xi_{n+p+1}\}, \quad (1)$$

where p and n are the order of basis function and the number of control points, respectively. The B-spline basis functions are defined, recursively, as

$$N_i^0(\xi) = \begin{cases} 1 & \text{if } \xi_i \leqslant \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases}, \ (p = 0) \quad (2)$$

and

$$N_i^p(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_i^{p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1}^{p-1}(\xi), \ (p = 1, 2, 3, \ldots). \quad (3)$$

Using the B-spline basis function $N_i^p(\xi)$ and weight w_i, the NURBS basis function $R_i^p(\xi)$ is defined as

$$R_i^p(\xi) \equiv \frac{N_i^p(\xi) w_i}{\sum_{j=1}^{n} N_j^p(\xi) w_j}. \quad (4)$$