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Inverse heat transfer analysis of multi-layered tube using thermal resistance network and Kalman filter

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ABSTRACT

In this study, a hollow cylindrical tube with a coating layer on its inner wall was considered. The thermal resistance network method was employed to solve the heat conduction in the tube. Unknown heat flux on the inner wall of the tube was estimated from measured temperature on the outer wall of the tube by the recursive input estimation algorithm consisting of the Kalman filter and real-time least squares. Assuming various operating conditions, the performance of the model developed in this study was evaluated.

1. Introduction

Heating of a tube can induce some adverse effects on system performance. Especially, combustion of a propellant is a major factor for melt, crack, erosion, and wear of the tubes at high pressures and temperatures. Therefore, it is important to know temperature distribution in the tube wall. If all boundary conditions are specified, then the temperature distribution in the tube wall can be obtained by solving the direct heat conduction problem (DHCP). However, if the measurement of physical parameters such as temperature and heat flux on one of the boundaries of a system is not possible as in the case of propellant combustion in a tube, then the system consisting of the tube cannot be analyzed by utilizing models to solve the direct heat conduction problems. Instead, estimating unknown temperature and heat flux on one boundary of the system from known conditions on other boundaries of the system can be performed by solving the inverse heat conduction problem (IHCP).

Analytical or numerical studies have been conducted to solve the inverse heat conduction problems. Analytic solutions were derived by using the integral or Laplace transform technique [1–5]. The analytic solutions are very efficient in the view of computation and are of fundamental importance for investigating basic properties, but are limited to simple geometries. Numerical methods including the sequential estimation technique were developed [6–9]. The conjugated gradient method was shown to be a straightforward and powerful iterative technique for solving linear and nonlinear inverse problems of parameter estimation [10–14]. Due to the necessity to estimate the history of unknown properties in real time in engineering applications, the recursive input estimation algorithm of digital estimation theory was proposed based on the concept of the Kalman filter technique and the least-squares estimation of recursive processing [15–18]. The Kalman filter is a set of mathematical equations providing an efficient computational solution of the least-squares method. The Kalman filter technique is simple and efficient, takes explicit measurement uncertainty incrementally, and can consider a priori information. In addition, if the physical system can be modeled, the Kalman filter and recursive least-squares algorithm is efficient for solving the problems with complex geometries.

The system modeling is required for the inverse heat conduction analysis. In previous studies, the system modeling was conducted using the finite element method [19–22]. The finite element method expresses approximate functions from unknown variables and determines small element values using the weighted residual method. The finite element method is suitable for solving problems with complex boundaries, but has disadvantage that numerical cost is high because of large amount of computation. In the meantime, the finite differential method expresses derivative terms using Taylor series and thus has advantage that numerical cost is relatively low. Therefore, in an effort to reduce the

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Nomenclature

A	surface area, m^2
B	sensitivity matrix
$[C]$	capacitance matrix
C_p	specific heat, J/kg K
E	total element number
$\{F\}$	thermal load vector
h	convectional heat transfer coefficient, W/m^2 s
H	measurement matrix
I	identity matrix
k	thermal conductivity, W/m K
K	Kalman gain
K_b	steady-state correction gain
m	element number for chrome layer
M	sensitivity matrix
$[M]$	conductance matrix
n	element number for steel layer
O	order of error
P	filter's error covariance matrix
P_b	error covariance matrix
q	heat flux, W/m^2
$\hat{q}$	estimated input vector, W/m^2
Q	process noise covariance
r	radius, m
R	measurement noise covariance
s	innovation covariance
t	time, s
T	temperature, K
T_∞	ambient temperature, K
T_0	initial temperature, K
V	element volume, m^3
X	state vector
$\bar{X}$	input estimator
Z	observation vector
$\bar{Z}$	bias innovation

Greek letters

α	thermal diffusivity, m^2/s
β	impulse duration time, s
γ	forgetting factor
δ	time interval between impulses, s
Γ	input matrix
θ	time-stepping parameter
κ	time (discretized), s
Λ	coefficient matrix
ν	measurement noise vector
ρ	density, kg/m^3
σ	standard deviation
Φ	state transition matrix
ω	process noise vector

Superscripts

n	time domain

Subscripts

c	chrome
C	convection
H	heat flux
i	inner
l	interface between chrome and steel layers
N	element number
o	outer
s	steel

amount of computation, the thermal resistance network method, which is based on the energy balance for control volumes, was used in heat transfer analysis [23,24].

In this study, a numerical model was developed to predict unknown heat flux on the inner wall of a tube from known temperature on the outer wall of the tube. The thermal resistance network method was employed to solve heat conduction in the tube. Using the recursive input estimation algorithm consisting of the Kalman filter and the real-time least squares, the unknown heat flux was estimated. Then, the validity of the model was evaluated by assuming various operating conditions.

2. Model description

2.1. Multi-layered tube problem

Fig. 1 shows a schematic of the cross-section of a hollow cylindrical tube consisting of chrome layer and steel layer, that is, the inner surface of the steel tube was considered to be coated with a thin chrome layer to prevent erosion and wear. The outer wall of the steel tube was presumed to be exposed to the ambient air. It was assumed that the outer wall temperature was given whereas the heat flux on the inner wall of the tube was unknown. Heat was assumed to be transferred mostly in the radial direction and the heat flux was considered to change with time. Therefore, the following conditions were assumed to simplify the problem.

- One-dimensional, axi-symmetric, and transient heat conduction.
- Constant properties (density, thermal conductivity, specific heat).
- Negligible contact resistance between chrome layer and steel layer.
- Constant convectional heat transfer coefficient outside the tube.
- Uniform heat flux on the inner wall of the tube.

Based on these assumptions, the governing equations, boundary conditions, and an initial condition were expressed as follows.

- Governing equations:

$$\frac{\partial^2 T(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial T(r,t)}{\partial r^2} = \left(\frac{1}{\alpha_c}\right)\frac{\partial T(r,t)}{\partial t}, \quad r_i \leqslant r \leqslant r_l \tag{1}$$

$$\frac{\partial^2 T(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial T(r,t)}{\partial r^2} = \left(\frac{1}{\alpha_s}\right)\frac{\partial T(r,t)}{\partial t}, \quad r_l < r \leqslant r_o \tag{2}$$

- Boundary conditions:

$$-k_c\frac{\partial T(r,t)}{\partial r} = q(t), \quad r = r_i \tag{3}$$

$$k_c\left(\frac{\partial T(r,t)}{\partial r}\right)_c = k_s\left(\frac{\partial T(r,t)}{\partial r}\right)_s, \quad r = r_l \tag{4}$$

$$-k_s\frac{\partial T(r,t)}{\partial r} = h(T(r,t) - T_\infty), \quad r = r_o \tag{5}$$

- Initial condition:

$$T(r,t) = T_0, \quad t = 0, \quad r_i \leqslant r \leqslant r_o \tag{6}$$