International Journal of Heat and Mass Transfer 85 (2015) 375–389

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A numerical method applied to forced and natural convection flows over arbitrary geometry

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ARTICLE INFO

Article history:
Received 20 December 2013
Received in revised form 28 January 2015
Accepted 1 February 2015

Keywords:
Building-cube method
Gridless approach
Preconditioning method
Computational cost

ABSTRACT

In this study, a numerical method based on Cartesian mesh method and gridless approach applied to forced and natural convection flows over arbitrary geometry is proposed. In the proposed method, a preconditioning method developed by the authors has been coupled with the building-cube method, owing to its simplicity and efficiency in mesh generation, implementation of solution algorithm, and post-processing. A gridless approach is further introduced as a wall boundary treatment for its adaptability to complex physics and arbitrary geometry problems. In this study, first, two classical cases of steady forced convection flow past a circular cylinder and natural convection flow over a circular cylinder are tested; the advantages of the gridless approach over the immersed boundary method are clearly shown. Then, further tests of unsteady forced convection flow past two side-by-side circular cylinders and natural convection flow over a Y-shaped fin are also conducted, which show that the present numerical method is also qualified for flow problems involving complex physics and arbitrary geometries. In addition, computational cost comparisons show that the time cost increase caused by the introduction of the gridless approach is within 20% for all test cases considered.

1. Introduction

Computational fluid dynamics (CFD) has achieved significant progress in recent years. In general, different numerical methods are used for flow problems involving compressible and incompressible fluids and to solve incompressible fluid flow problems by methods designed for compressible ones leads to bad convergence originating from the stiffness of the solution. As a result, a preconditioning method has been proposed by Choi and Merkle [1], making it possible to solve compressible and incompressible fluid flow problems with a single method. Our research group has developed the preconditioning method based on previous research for flow problems with grid points in general curvilinear coordinates [2]. However, it still has a crucial issue how practical flow problems with arbitrary geometry are simulated.

There are choices of different mesh generation approaches, such as structured mesh, unstructured mesh and Cartesian mesh method. The building-cube method (BCM), a block-structured Cartesian mesh method, has been proposed by Nakahashi [3] as a next-generation CFD candidate because of its simplicity and efficiency. The mesh generation process of BCM has been proved to be completely automatic and robust for complex geometry configurations. However, wall boundary treatment is a critical issue for the Cartesian mesh method, because grid points are generally not located on the solid wall surface.

There are also different wall boundary treatments for the Cartesian mesh method, such as cut-cell [4], volume of fluid [5], and the immersed boundary (IB) method. The IB method has been tested with BCM in previous research [6]; it was demonstrated to be a good choice for curved surface representation as well as for estimation of pressure and velocity of the wall regions. However, since the IB method is essentially based on the original Cartesian mesh, an extremely fine mesh is required for problems with a significant gradient in the wall boundary region. An alternative solution is to use a hybrid method incorporating the Cartesian mesh method and gridless approach. In the hybrid method, the Cartesian mesh method is used as a background grid for resolution of the entire flow field and the gridless approach is used as a wall boundary treatment for resolution of regions close to the wall boundaries.

In this study, a hybrid method comprising the Cartesian mesh method and the gridless approach is proposed based on the preconditioning method and is applied to the simulation of forced and natural convection flows over arbitrary geometry. In the hybrid method, the Cartesian mesh method is used as a background grid owing to its simplicity and efficiency, and the gridless approach is used as a wall boundary treatment because of its adaptability to complex physics and arbitrary geometries. First,

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2015.02.001
0017-9310/

two classical cases of steady forced convection flows past a circular cylinder and natural convection flows over a circular cylinder are tested to show the difference of the IB method and the gridless approach as wall boundary treatments. Then, further tests of unsteady forced convection flows past two side by side circular cylinders and natural convection flows over a Y-shaped fin are also conducted, which show the capability of the hybrid method for flow problems involving complex physics and arbitrary geometries. In addition, computational cost comparisons are also shown to further evaluate the gridless approach in the present method.

2. Numerical methods

2.1. Governing equation

The governing equation used in this study is based on the preconditioned two-dimensional (2-D) compressible Navier–Stokes equations. The vector form equation set in general curvilinear coordinates is written as

$$\Gamma\frac{\partial \hat{\mathbf{Q}}}{\partial t}+\frac{\partial \mathbf{F_i}}{\partial \xi_i}+\frac{\partial \mathbf{F_{vi}}}{\partial \xi_i}=0. \tag{1}$$

where the conservative vector $\boldsymbol{Q}$, inviscid flux $\boldsymbol{F_i}$ and viscous flux $\boldsymbol{F_{vi}}$ are defined as

$$\hat{\mathbf{Q}}=J\begin{bmatrix}p\\u_1\\u_2\\T\end{bmatrix},\quad \mathbf{F_i}=J\left(\frac{\partial \xi_i}{\partial x_j}\right)\begin{bmatrix}\rho u_j\\ \rho u_1 u_j+\delta_{j1}p\\ \rho u_2 u_j+\delta_{j2}p\\ (e+p)u_j\end{bmatrix},$$
$$\mathbf{F_{vi}}=-J\left(\frac{\partial \xi_i}{\partial x_j}\right)\begin{bmatrix}0\\ \tau_{j1}\\ \tau_{j2}\\ \tau_{jk}u_k+\kappa\partial T/\partial x_i\end{bmatrix}. \tag{2}$$

The preconditioning method is based on the one proposed by Weiss and Smith [7] and the preconditioning matrix Γ is defined as:

$$\Gamma=\begin{bmatrix}\theta & 0 & 0 & \rho_T\\ \theta u_1 & \rho & 0 & \rho_T u_1\\ \theta u_2 & 0 & \rho & \rho_T u_2\\ \theta h-(1-\rho h_p) & \rho u_1 & \rho u_2 & \rho_T h+\rho h_T\end{bmatrix}. \tag{3}$$

θ and U_r are the preconditioning parameter and the switching parameter defined as

$$\theta=\frac{1}{U_r^2}-\frac{\rho_T(1-\rho h_p)}{\rho h_T},\quad U_r=\begin{cases}\varepsilon c & (u<\varepsilon c)\\ u & (\varepsilon c<u<c\\ c & (c<u)\end{cases} \tag{4}$$

The governing equation is solved by a finite-difference scheme in the background grid of Cartesian mesh.

For spatial discretization, the preconditioned Roe scheme [2] is used for inviscid flux estimation, and a second-order MUSCL scheme is used for left and right state primitive variable evaluation. For time integration process, the preconditioned implicit LU-SGS scheme [2] is used.

For thermophysical properties estimation in the present study, all mathematical models are referred to from PROPATH [8] as external functions.

2.2. Building-cube method

In this study, BCM proposed by Nakahashi et al. is used as the background grid generation approach for CFD calculation [3]. In BCM, blocks called "Cube" are used as basic calculation units to resolve the computational domain. The uniform Cartesian mesh element called 'Cell' is used as calculation grid in the cubes, as is shown in Fig. 1. The basic advantage of BCM is its simplicity in mesh generation for complex geometry configuration problems. The simplicity will be more significant for massively parallel CFD applications in the future [9].

Overlapping cells are used for data communication between adjacent cubes. Bilinear-interpolation (extrapolation) is used to estimate the values of any physical properties at the overlapping cells. The wall boundary is defined in a staircase manner [10,11] in BCM for algorithm simplicity, with flags representing wall cells and fluid cells to distinguish the solid wall surface, as is shown in Fig. 2.

Since BCM is a Cartesian mesh method, wall boundary treatment is a critical issue. In the wall boundary regions, the significant physical gradients are mostly in the wall normal directions. While in the Cartesian mesh method, the grid resolution is the same in all directions, thus an extremely fine mesh is to be used for complex physics problems with a significant gradient in the wall boundary regions, which might be unacceptable for certain cases. The IB method was proved to be of superior fidelity in solid wall surface representation in the previous study [6], but it is still based on the Cartesian mesh. An alternative solution is the hybrid method of Cartesian mesh method and gridless approach. In the gridless approach, it is flexible to use a fine grid resolution in the wall normal direction with a coarse grid resolution in the other direction.

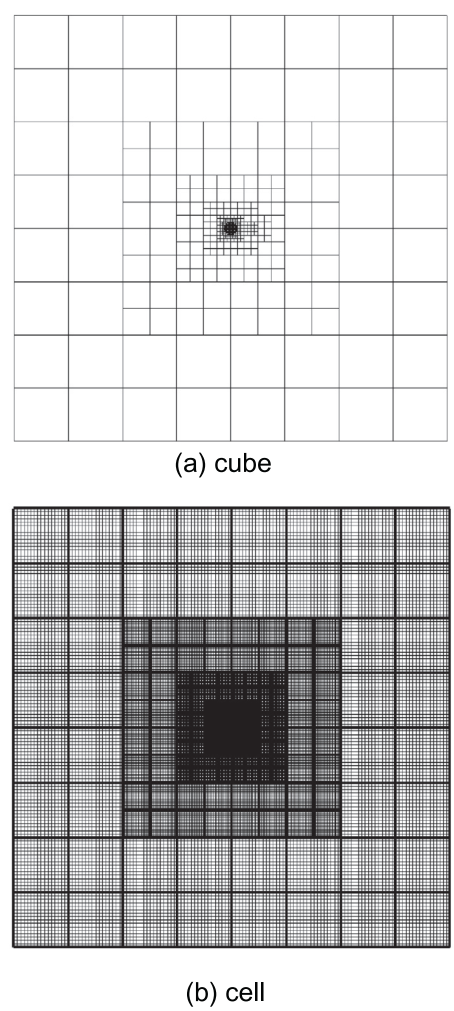

Fig. 1. Schematic of the mesh configuration for the BCM.