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# Constant wall heat flux boundary condition in micro-channels filled with a porous medium with internal heat generation under local thermal non-equilibrium condition

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ABSTRACT

Forced convection heat transfer in a micro-channel filled with a porous material saturated with rarefied gas with internal heat generation is studied analytically in this work. The study is performed by analysing the boundary conditions for constant wall heat flux under local thermal non-equilibrium (LTNE) conditions. Invoking the velocity slip and temperature jump, the thermal behaviour of the porous-fluid system is studied by considering thermally and hydrodynamically fully-developed conditions. The flow inside the porous material is modelled by the Darcy–Brinkman equation. Exact solutions are obtained for both the fluid and solid temperature distributions for two primary approaches models A and B using constant wall heat flux boundary conditions. The temperature distributions and Nusselt numbers for models A and B are compared, and the limiting cases resulting in the convergence or divergence of the two models are also discussed. The effects of pertinent parameters such as fluid to solid effective thermal conductivity ratio, Biot number, Darcy number, velocity slip and temperature jump coefficients, and fluid and solid internal heat generations are also discussed. The results indicate that the Nusselt number decreases with the increase of thermal conductivity ratio for both models. This contrasts results from previous studies which for model A reported that the Nusselt number increases with the increase of thermal conductivity ratio. The Biot number and thermal conductivity ratio are found to have substantial effects on the role of temperature jump coefficient in controlling the Nusselt number for models A and B. The Nusselt numbers calculated using model A change drastically with the variation of solid internal heat generation. In contrast, the Nusselt numbers obtained for model B show a weak dependency on the variation of internal heat generation. The velocity slip coefficient has no noticeable effect on the Nusselt numbers for both models. The difference between the Nusselt numbers calculated using the two models decreases with an increase of the temperature jump coefficient.



## 1. Introduction

The study of microscale heat transfer has attracted significant interests over the last decade leading to the miniaturisation of various technological devices such as pumps, turbines, mixers and heat pipes, which are generally referred to as micro-flow devices (MFDs) [1,2]. Such micro-devices have revolutionised complex systems for medical diagnosis and surgery, chemical analysis, biotechnology and electronic cooling [2]. The flow regimes and modelling of flow in micro-systems are classified using the Knudsen number ($Kn = \lambda / D_H$), which is defined as the ratio of the molecular mean-free-path ($\lambda$) to a characteristic macroscopic length scale, i.e. the hydraulic diameter ($D_H$). It allows having a measure of the validity of the continuum model and a classification of gas flow regimes [2,3]. The Navier–Stokes equations, which assume the continuum flow, work well with the no-slip conditions at $Kn < 0.001$. The continuum assumption is still valid when $0.001 < Kn < 0.1$, while a finite slip condition needs to be considered at the boundary of the flow domain (e.g. [4–9]). The regime of flow with $0.001 < Kn < 0.1$ is called slip-flow regime. At higher Knudsen numbers, the Navier–Stokes equation is not applicable and the kinetic theory must be applied [2,3]. Modelling convection through such small devices is different from its macroscale counterparts in that the velocity slip and temperature jump are included, as noted in [10]. This article focuses on the slip-flow regime in a channel filled with a porous material. Analysis of heat and fluid flow in micro-channels filled with a porous material under local thermal equilibrium condition has been studied extensively (e.g. [7,11,12]). However, analytical studies on slip flow in porous-saturated

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## Nomenclature

| | |
|---|---|
| $a_{sf}$ | interfacial area per unit volume of porous media ($m^{-1}$) |
| $A$ | constant parameter defined by Eq. (29-3) |
| $B$ | constant parameter defined by Eq. (29-4) |
| $Bi$ | Biot number, $\frac{a_{sf}h_{sf}h_0^2}{(1-\varepsilon)k_s}$ defined by Eq. (21) |
| $C$ | constant parameter defined by Eq. (29-8) |
| $c_p$ | specific heat of the fluid, ($J\,kg^{-1}\,K^{-1}$) |
| $Da$ | Darcy number, $K/H^2$ |
| $D_H$ | hydraulic diameter of the channel ($4H$) |
| $G_1$, $G_2$ and $G_3$ | constant parameters defined by Eqs. (29-5)–(29-7), respectively |
| $h_{sf}$ | fluid to solid heat transfer coefficient ($W\,m^{-2}\,K^{-1}$) |
| $2H$ | height of the micro-channel (m) |
| $H_1$ and $H_2$ | constant parameters defined by Eqs. (29-9) and (29-10), respectively |
| $J_1$, $J_2$ and $J_3$ | constant parameters defined by Eqs. (51-1), (51-2) and (51-4), respectively |
| $K$ | permeability of the porous medium ($m^2$) |
| $k$ | the ratio of fluid effective thermal conductivity to that of the solid, $(\varepsilon k_f)/(1-\varepsilon)k_s$ defined by Eq. (21) |
| $k_f$ | thermal conductivity of the fluid ($W\,m^{-1}\,K^{-1}$) |
| $k_{f,eff}$ | effective thermal conductivity of the fluid, $\varepsilon k_f$ |
| $k_s$ | thermal conductivity of the solid ($W\,m^{-1}\,K^{-1}$) |
| $k_{s,eff}$ | effective thermal conductivity of the solid, $(1-\varepsilon)k_s$ |
| $Kn$ | Knudsen number based on permeability, $\lambda/\sqrt{K}$ |
| $M$ | viscosity ratio $\mu_{eff}/\mu$ |
| $Nu$ | Nusselt number |
| $p$ | pressure (Pa) |
| $Pr$ | Prandtl number |
| $q$ | heat flux ($W\,m^{-2}$) |
| $S_f$ | internal heat generation within the fluid phase $W\,m^{-3}$ |
| $S_s$ | internal heat generation within the solid phase $W\,m^{-3}$ |
| $T$ | temperature (K) |
| $T_1$, $T_2$ | constant parameters defined by Eqs. (51-5) and (51-7), respectively |
| $T_{f,m}$ | average temperature (K) |
| $u$ | longitudinal velocity (m/s) |
| $\bar{u}$ | average velocity |
| $u_r$ | characteristic velocity, $-(H^2/\mu)(\partial p/\partial x)$ |
| $U$ | dimensionless velocity, $u/u_r$ |
| $\bar{U}$ | dimensionless average velocity |
| $w$ | $=w_f+w_s$ |
| $w_f$ | parameter defined by Eq. (22) $=(H/q_w)S_f$ |
| $w_s$ | parameter defined by Eq. (22) $=(H/q_w)S_s$ |
| $W$ | constant parameter defined by Eqs. (51-6) |
| $x$ | longitudinal coordinate (m) |
| $X$ | constant parameter defined by Eqs. (51-3) |
| $y$ | transverse coordinate (m) |
| $Y$ | dimensionless $y$ coordinate, $y/H$ |
| $Z$ | constant parameter, $\sqrt{1/MDa}$ |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | velocity slip coefficient $=\alpha_0\frac{2-\sigma_v}{\sigma_v}Kn\sqrt{Da}$ defined by Eq. (10) |
| $\beta$ | temperature jump coefficient $=\beta_0\frac{2-\sigma_T}{\sigma_T}\frac{2\gamma}{\gamma+1}\frac{Kn}{\Pr}\sqrt{Da}$ defined by Eq. (20) |
| $\gamma$ | specific heat transfer ratio |
| $\delta$ | $=\sqrt{Bi(1+1/k)}$, defined by Eq. (29-1) |
| $\varepsilon$ | porosity of the porous medium |
| $\Delta\theta$ | dimensionless temperature difference $=\theta_f-\theta_s$ |
| $\theta$ | dimensionless temperature |
| $\theta_{f,m}$ | dimensionless bulk fluid temperature defined by Eq. (34) |
| $\lambda$ | mean free path m |
| $\mu$ | viscosity ($kg\,m^{-1}\,s^1$) |
| $\mu_{eff}$ | effective viscosity of the porous medium ($kg\,m^{-1}\,s^1$) |
| $\rho$ | density, ($kg/m^3$) |
| $\sigma$ | accommodation coefficient |
| $\zeta$ | parameter used in Eq. (13) $=1+\alpha Z\tanh(Z)$ |
| $\chi$ | parameter used in Figs. 14 and 15 $=(\theta\vert_{\beta=0.1}-\theta\vert_{\beta=0})/\theta\vert_{\beta=0.1}$ |

*Subscripts*

| | |
|---|---|
| $eff$ | effective property |
| $f$ | fluid |
| $m$ | mean |
| $s$ | solid |
| $T$ | thermal |
| $v$ | momentum |
| $w$ | wall |

*Superscripts*

| | |
|---|---|
| $-$ | mean value |
| $'$,$''$,$'''$,$''''$ | first, second, third, and forth derivatives with respect to $Y$ |

micro-channels under local thermal non-equilibrium condition have not been conducted to the same extent.

In principle, there are two methods of modelling the energy equation in a porous medium. These are the local thermal equilibrium (LTE) and local thermal non-equilibrium (LTNE) models [13]. LTE model holds only when the temperature difference between the solid and fluid phases is negligibly small. In reality, however, this temperature difference may not be small. Hence, a more precise analysis should relax the assumption of LTE and use LTNE model instead. However, the use of LTNE model in a channel subject to a constant wall heat flux boundary condition requires additional information to account for the modes of energy communication between the two phases at the channel wall [14,15]. This information are usually provided in the form of models related to the constant wall heat flux boundary conditions [14,15]. This, in turn, makes the thermal behaviour of the system dependent upon the applied model. Extra levels of complexity are hence added to the problem, which involve devising the proper models and including them in the analysis. In contrast, boundary conditions for constant wall temperature are clear and both phases have temperatures that are equal to the wall temperature.

One of the early studies was the numerical analysis of natural convection heat transfer in a vertical open-ended parallel-plate micro-channel filled with porous media by Haddad et al. [12] under LTNE condition using the Darcy–Brinkman–Forchheimer model. It was observed that the Nusselt number decreased with increase in the $Kn$ number, Darcy number and thermal conductivity ratio. Further, the authors reported that the Nusselt number increased as Forchheimer number and Biot number increased. In another study, Haddad et al. [16] performed numerical investigations into forced convection inside a micro-channel assuming LTE condition and the Darcy–Brinkman–Forchheimer model. They found that the rate of heat transfer increased as the Darcy number increases and it decreased when the Knudsen number or Forchheimer number increases. Haddad et al. [17] studied numerically the laminar forced convection gaseous slip-flow through parallel-plates micro-channel filled with porous medium under LTNE condition and assuming Darcy–Brinkman–Forchheimer model. The same problem was studied numerically by Haddad et al. [18] in a circular micro-channel under LTE condition. In both studies, the micro-channel wall was subjected to constant wall temperature condition. Similar to their previous studies, they found that the